Global Invariants in Line Geometry

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As known the geometry of a trajectory surfaces tracing by an oriented line (spear) is important in line geometry and spatial kinematics. Until early 1980s, although two real integral invariants, the pitch of angle λ_x and the pitch ℓ_x of an x-trajectory surface were known, any dual invariant of the surface were not. Because of the deficiency, line geometry wasn't being sufficiently studied by using dual quantities.

In this presentation, a dual integral invariant Λ_x (the dual angle of pitch of an x- trajectory surface) are introduced and shown that $\Lambda_x = \lambda_x - \varepsilon\ell_x$ [1]. This new invariant gives $\Lambda_x = 2\pi - A_x$ or $\lambda_x = 2\pi - a_x$ and $\ell_x = -\iint(\delta_u + \delta_v)dudv$ which have the new geometric interpretations of x-trajectory surface where a_x is the measure of the spherical area on the unit sphere described by the generator of x- closed trajectory surface and δ_u and δ_v are the distribution parameters of the principal surfaces of the $X(u, v)$ - closed congruence.

Therefore all the relations between the global invariants λ_x, ℓ_x, a_x, K, T, σ, and s_1 of x- c.t.s. are worth reconsidering in view of the new geometric explanations. Thus some new results and new explanations are gained [2,3,4,5].

Furthermore, as a limit position of the surface, some new theorems and comments related to space curves are obtained. Also the proofs of the theorems of Jacobi and Fenchel are given by an elegant methods.

References

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