

How Many Affine Connections Exist in General Dimension?

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ABSTRACT

The question is, more precisely, how many real analytic affine connections exist locally on a manifold M of dimension n ? The main tool is the Cauchy-Kowalevski Theorem. Our question is answered for torsion-free connections, or connections with full torsion, or for equiaffine connections, respectively. Also, the same question is answered if we suppose, in addition, the Ricci tensor to be symmetric, or skew-symmetric, respectively. The answer is always done by some cubic polynomial in variable n . The most interesting result says that, for n tending to infinity, the additional assumption for the Ricci tensor is not too restrictive. Namely, if we denote by $\text{con}(n)$, $\text{skew_Ric-con}(n)$ and $\text{sym_Ric-con}(n)$, respectively, the corresponding polynomials, then the fractions (rational functions in n) $\text{skew_Ric-con}(n)/\text{con}(n)$ and $\text{sym_Ric-con}(n)/\text{con}(n)$ are tending to 1 for n tending to infinity. This seeming paradox can be explained by the fact that the operation of calculating the Ricci tensor from the Christoffel symbol is a nonlinear operation