Kepler Problem and Formally Real Jordan Algebras

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Abstract

The goal of this lecture course is to present a general mathematical theory behind the Kepler problem, i.e., the mathematical model for the simplest solar system. This theory is based on the formally real Jordan algebras introduced by Pascual Jordan in 1933 in his attempt to give a better formulation of quantum mechanics.

The Kepler problem is the mathematical model whose solution gives an explanation of the Kepler's Laws for planetary motion. Its quantum version, refereed to as the Coulomb problem, is the mathematical model whose solution gives an explanation of the Rydberg formula for the spectral lines of the hydrogen gas. There is no doubt that this problem has been crucially important to the development of fundamental physics.

It is a usual case that physicists study a particularly nice example, and mathematicians sort out the beautiful theory behind the example. One might paraphrase this by saying that physicists open the door and mathematicians make an extensive search of the house for gold.

Since the Kepler problem is certainly a nice example by almost any criterion, it is worth sorting out the beautiful mathematical theory behind it. Following the advice of "thinking deeply on simple things", we found that [5, 6, 7] the secret of Kepler problem lies in the formally real Jordan algebras [2, 3, 1]. Once this secret is revealed, it is clear that the Kepler problem belongs to a large family of super integrable systems which is much larger than any one had previously imagined, moreover, a new discovery [8] concerning the MICZ-Kepler problems [4, 9] and the Lorentz transformations is quickly obtained.

The primary goal in this set of lectures is to share the aforementioned secret with the audiences who are assumed to have some rudimentary knowledge in differential geometry. The topics that we shall cover are listed below:

References

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