Constant Mean Curvature Surfaces with Boundary

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1. Abstract

Surfaces with constant mean curvature (cmc) in Euclidean space are mathematical models in physics of interfaces and capillary phenomena. For example, in the absence of gravity, the shape of an interface M in physical equilibrium is given by the Laplace equation $P_{ext} - P_{ins} = 2H\gamma$, which measures the difference of pressures between the exterior P_{ext} and the inside P_{ins} in terms of the mean curvature H of M and the surface tension γ of the interface.

Two classical results, namely, the Hopf and the Alexandrov theorems, characterize the spheres in the family of cmc closed surfaces. However, if we now assume that the boundary of the surface is a round circle, it is unknown up today if planar round disks or spherical caps are the only cmc compact surfaces with circular boundary. In these lectures, we study the shape of a cmc compact surface with non-empty boundary and we are concerned in what type of geometric information can be derived about the shape of the surface in terms of the prescribed boundary.

The objective of this course is twofold. First, give a state-of-art of the main results and the open problems in the theory of cmc compact surfaces with boundary. Second, familiarize with the techniques employed in this theory: tangency principle, Alexandrov reflection process, flux formula, Dirichlet problem.

2. Outline

Lecture 1: Introduction and motivation.

- Interfaces and capillarity.
- Historical introduction.
- Examples.
- Closed surfaces: classical theorems.

Lecture 2: The tangency principle.

- The maximum principle and the comparison principle.

- Applications.
- The Alexandrov reflection method.

Lecture 3: CMC compact surfaces with boundary.

- Motivation and problems.
- The flux formula.
- The circular case.

Lecture 4: The Dirichlet problem.

- The method of continuity.
- Height and gradient estimates.
- Results of existence.

Lecture 5: CMC surfaces in other ambient spaces.

- The hyperbolic space.
- The Lorentz-Minkowski space.

References

[1] R. Finn, Equilibrium capillary surfaces. Grundlehren der Mathematischen Wissenschaften, 284. Springer-Verlag, New York, 1986.
[2] D. Gilbarg, N. Trudinger, Elliptic partial differential equations of second order. Second edition. Grundlehren der Mathematischen Wissenschaften, 224. Springer-Verlag, Berlin, 1983.
[3] K. Kenmotsu, Surfaces with Constant Mean Curvature. Translations of Mathema- tical Monographs, AMS, Providence, Rhode Island, 2003.
[4] R. López, Constant Mean Curvature Surfaces with Boundary. Springer Monographs in Mathematics. Springer Verlag. Berlin, 2013.