Unified Variational Structures for Planar Rods with General Distributed Energies

Yimu Mao

Stiftung Louisenlund, 24357 Güby, Germany

We present a unified variational framework for planar elastic rods subject to arbitrary distributed loads along their length. By systematically transforming nested energy integrals into single-integral expressions, the approach unifies gravitational, magnetic, and other distributed forces in a compact, geometrically transparent form. The method highlights underlying variational and differential-geometric structures, and is analytically tractable for deriving governing equations and exploring planar rod mechanics. Applications to classical heavy elastica and hard-magnetic rods illustrate exact recovery of known results.