50 years of the problem the source of Kerr solution: Source of the Kerr-Newman solution as BPS-saturated bag-string-quark system.

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Based on:

A.B., Gravitating Lepton Bag Model, JETP, 2015 (to appear).

A.B., Emergence of the Dirac Equation in the Solitonic Source of the Kerr Spinning Particle, Gravitation and Cosmology, Vol. 21 (1), 28 (2015) [arXiv:1404.5947].

Black holes as elementary particles[G.'t Hooft (1990), A. Sen (1995), C.F.E. Holzhey and F. Wilczek (1992), A.Salam and J. Strathdee (1976)].

KERR-NEWMAN solution as Spinning Particle.

The experimentally observable parameters of an electron (mass m, spin J, charge e and magnetic moment μ) indicate that its gravitational and electromagnetic fields correspond to Kerr-Newman (KN) black hole solution!

Spin of electron is extremely high, $a = J/m >> m$ ($a/m \approx 10^{44}$), and black hole horizons disappear, corresponding to ULTRA-EXTREME KN solution. No horizons - naked topological defect:

SINGULAR RING *of Compton radius* – branch line of space-time, forming a door to a mirror world. *TWO-SHEETED space-time!*

CONFLICT BETWEEN GRAVITY AND QUANTUM THEORY:
REGULARIZATION of Kerr's source: GRAVITATING SOLITON
The requirements:

1) **Flat space-time INSIDE the soliton!**
2) **The exact Kerr-Newman solution OUTSIDE the soliton!**
3) **A smooth transition between internal and external metrics**

determine UNAMBIGUOUSLY structure of the source as a "Bag-String-Quark" system.

SHAPE OF the BAG is uniquely determined by the KN metric $g^{(KN)}_{\mu\nu} = \eta_{\mu\nu} + 2H_{(KN)}k_\mu k_\nu$, **where** $H_{(KN)} = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta}$.

and ZERO GRAVITY SURFACE: $H_{(KN)} = 0 \quad \Rightarrow \quad r = e^2/2m.$

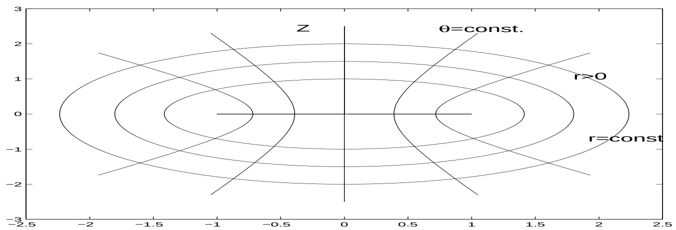
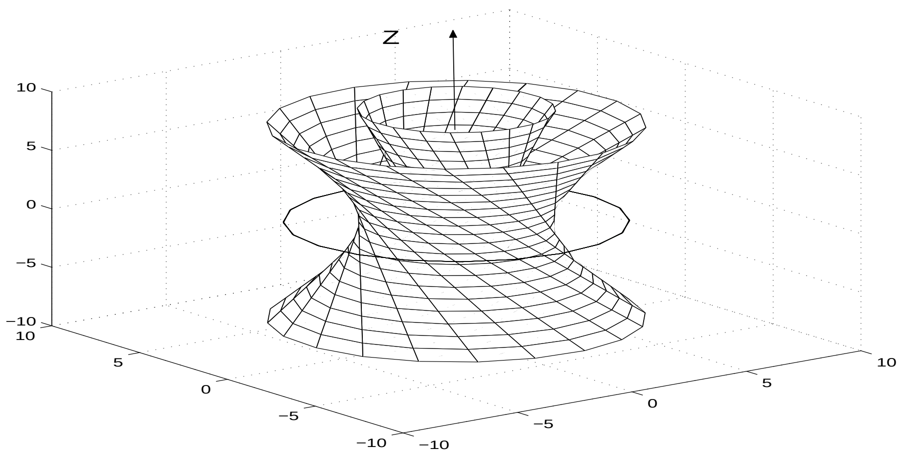


Figure 1: **Kerr's oblate spheroidal coordinates cover space-time twice.**



The metric and vector potential $A^\mu_{KN} = Re\frac{e}{r + ia\cos\theta}k^\mu$ **are collinear with Principal Null Directions** k^μ **controlled by the KERR THEOREM.**

PECULIARITIES:

1.- Shape of the KN soliton:

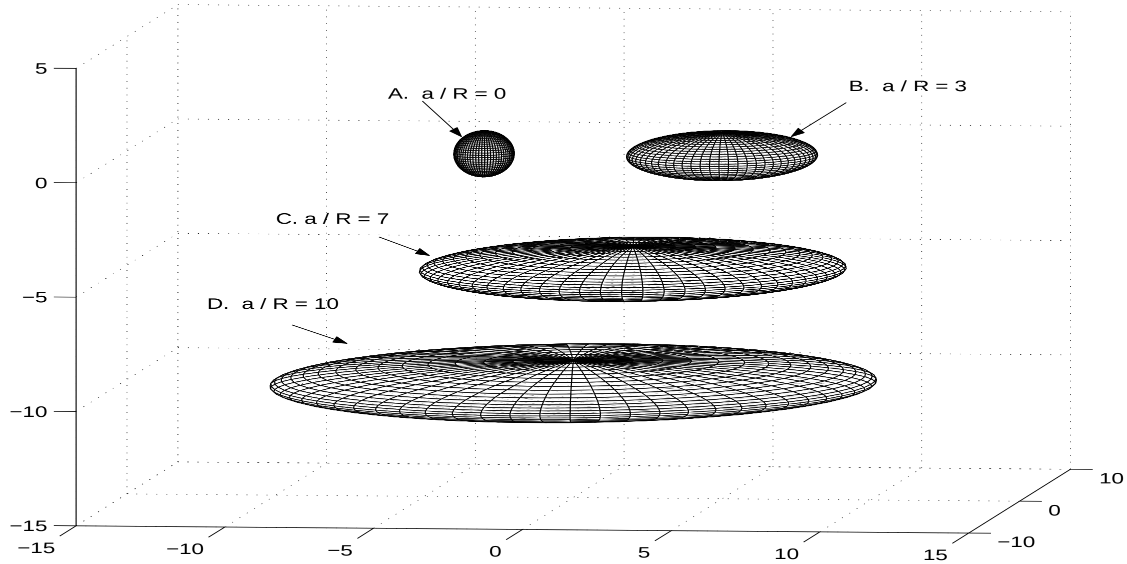


Figure 2: **The disk-like bubble for different ratio a/R.**

2.- The quartic potential $V(r) = \lambda(|\phi|^2 - \Phi^2)^2$, used in the known bag models and in the known Nielsen-Olesen string model, doesn't go. The KN-source requires *a few chiral fields* $\Phi^i(r), \; i = 1, 2, 3.$

3. - Twisted Kerr congruence k^μ is controlled by the KERR THEOREM which yields two solutions $k^{\mu\pm}$ which determine TWO-SHEETED metric, $g^\pm_{\mu\nu}$, vector potential $A^\mu_{KN} = Re\frac{e}{r+ia\cos\theta}k^{\mu\pm}$, and the associated Dirac field !

PECULIARITIES OF MIT-BAG AND KN-BAG MODELS.

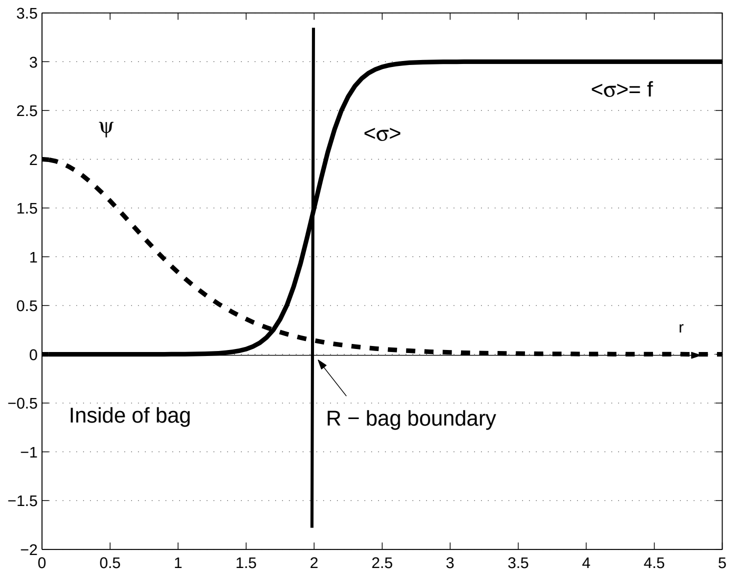


Figure 3: Illustration of the quark confinement in the bag models. Vacuum field σ is determined by quartic potential.

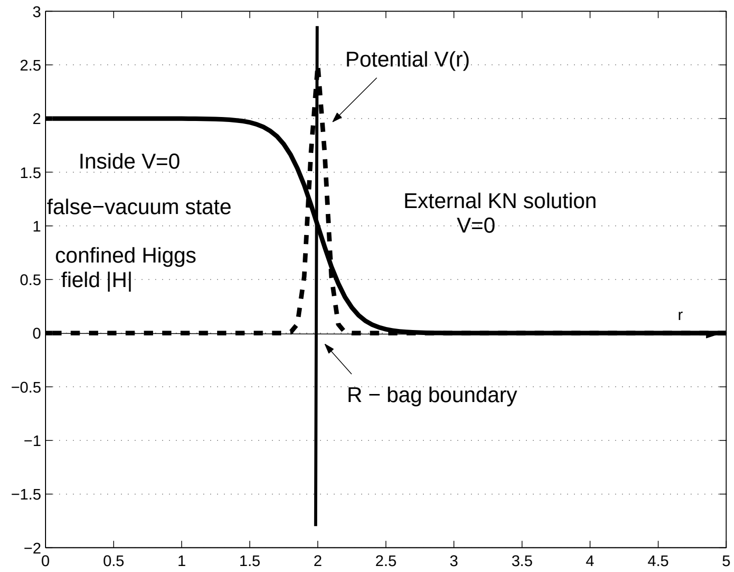


Figure 4: The KN soliton bag model (Q-ball). Potential $V(R)$ forms a narrow spike at the bag boundary. The Higgs field H is confined inside the bag forming a false-vacuum state.

KN-bag should have $V_{int} = V_{ext} = 0$, and a narrow spike at the bag-boundary.
Formation of such a potential requires *a few chiral fields* $\Phi^i(r), \ i = 1, 2, 3.$

Supersymmetric field model of phase transition.

Triplet of the chiral fields $\Phi^{(i)} = \{H, Z, \Sigma\}$, where H is Higgs field.

Lagrangian

$$\mathcal{L} = -\frac{1}{4}\sum_{i=1}^{3} F_{\mu\nu}^{(i)} F^{(i)\mu\nu} - \frac{1}{2}\sum_{i=1}^{3} (\mathcal{D}_{\mu}^{(i)}\Phi^{(i)})(\mathcal{D}^{(i)\mu}\Phi^{(i)})^{*} - V, \tag{1}$$

covariant derivatives $\mathcal{D}_{\mu}^{(i)} = \nabla_{\mu} + ieA_{\mu}^{(i)}$.

Superpotential

$$W = \Phi^{(2)}(\Phi^{(1)}\bar{\Phi}^{(1)} - \eta^{2}) + (\Phi^{(2)} + \mu)\Phi^{(3)}\bar{\Phi}^{(3)}, \tag{2}$$

determines the potential

$$V(r) = \sum_{i} |\partial_{i} W|^{2}, \tag{3}$$

$\mathcal{H} \equiv \Phi^{(1)}$ **is taken as Higgs field.**

Vacuum states $V_{(vac)} = 0$ **are determined by the conditions** $\partial_i W = 0$. **The model yields two vacuum solutions:**

(I) vacuum state inside the bag: $|\mathcal{H}| = \eta;\ Z = -\mu;\ \Sigma = 0,$

(II) external vacuum state: $|\mathcal{H}| = 0;\ Z = 0;\ \Sigma = \eta.$

The Higgs field $\mathcal{H}$ is confined inside the bag. Gauge symmetry is broken $\Rightarrow$ supersymmetric false vacuum state.

Basic equations for interaction of the electromagnetic and the Higgs field $\mathcal{H}(x) = |\mathcal{H}|e^{i\chi(x)}$ confined inside the bubble:

$$\mathcal{D}^{(1)}_\nu \mathcal{D}^{(1)\nu}\mathcal{H} = \partial_{\mathcal{H}^*} V, \tag{4}$$

$$\nabla_\nu \nabla^\nu A_\mu = I_\mu = \frac{1}{2} e|\mathcal{H}|^2(\chi,_\mu + eA_\mu). \tag{5}$$

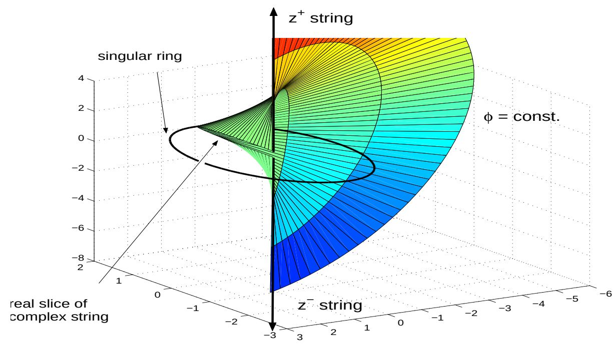


Figure 5: The Kerr surface $\phi = const$. The Kerr congruence is tangent to singular ring at $\theta = \pi/2$.

Peculiarities of the KN soliton model:

(i) closed flux **of the KN electromagnetic potential forms a *quantum Wilson loop* $\oint eA_\varphi d\varphi = -4\pi ma$, which results in** **quantization of the soliton spin**, $J = ma = n\hbar/2,\ n = 1, 2, 3, ...,$

(ii) the Higgs condensate forms a *coherent vacuum state* oscillating with the frequency $\omega = 2m$ – oscillons, Q-balls (G.Rosen 1968, Coleman 1985).

Supersymmety, Bogomolnyi bound and stability.

The Einstein-Maxwell eqs. are trivially satisfied inside and outside the bubble.
Inside the bubble and at the boundary metric is flat.
Domain Wall is not rotating and Hamiltonian is simplified

$$H^{(ch)} = T_0^{\ 0(ch)} = \frac{1}{2}\sum_{i=1}^{3} [\sum_{\mu=0}^{3} |\mathcal{D}_\mu^{(i)}\Phi^i|^2 + |\partial_i W|^2].$$

Although metric is flat, influence of gravity is saved in the shape of the bag and in the twisted form of the electromagnetic field.
We must use the adapted Kerr coordinate system $x+iy = (r+ia)e^{i\phi}\sin\theta, \quad z = r\cos\theta, \quad t = \rho - r$, **in which the KN vector potential takes the form**

$$A_\mu dx^\mu = -Re\ [(\frac{e}{r+ia\cos\theta})](dr - dt - a\sin^2\theta d\phi). \tag{6}$$

The terms $A_\phi d\phi$ **and** $A_t dt$ **may be separated by introducing the first order equations**

$$\mathcal{D}_t^{(1)}\Phi^1 = 0, \quad \mathcal{D}_\phi^{(1)}\Phi^1 = 0, \tag{7}$$

which lead to consequences (i) and (ii), and these derivatives drop out of the Hamiltonian.

The rest of the Hamiltonian is reduced to integral over one Kerr variable r.

$$H^{(ch)} = T_0^{\ 0(ch)} = \frac{1}{2}\sum_{i=1}^{3} \ [|\mathcal{D}_r^{(i)}\Phi^i|^2 + |\partial_i W|^2], \tag{8}$$

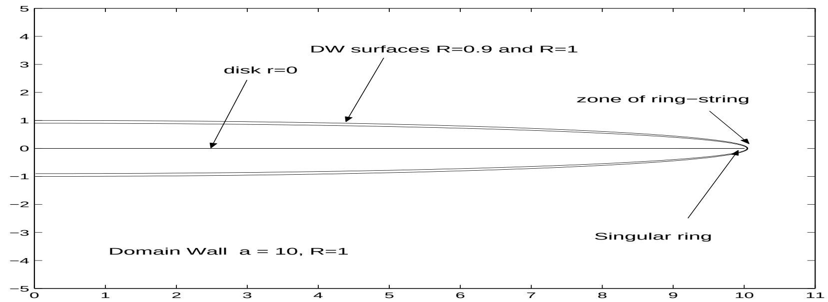


Figure 6: **Axial section of the spheroidal domain wall phase transition.**

Now one uses the suggested by Cvetiĉ & Rey *TRICK* allowing to transform it to Bogomolnyi form

$$H^{(ch)} = T_0^{\ 0(ch)} = \frac{1}{2}\sum_{i=1}^{3} \ [|\mathcal{D}_r^{(i)}\Phi^i - e^{i\chi_i}\partial_i\bar{W}|^2 + 2Re\ e^{-i\chi_i}\partial_i\bar{W}\mathcal{D}_r^{(i)}\Phi^i] \tag{9}$$

The angles χ_i are determined by phase of the oscillating Higgs field

$$\Phi(x) \equiv \Phi^1(x) = |\Phi^1(r)|e^{i\chi(t,\phi)}. \tag{10}$$

They should be independent from r, and be chosen to cancel the square terms. It yields $\chi_1 = 2\chi(t,\phi), \ \chi_2 = \chi_3 = 0.$

We obtain the first order Bogomolnyi equations

$$\mathcal{D}_r^{(i)}\Phi^i = \partial W/\partial\Phi^i, \quad \mathcal{D}_r^{(i)}\bar{\Phi}^i = \partial\bar{W}/\partial\bar{\Phi}^i. \tag{11}$$

The KN source forms the stable BPS-saturated configuration.

Hamiltonian turns into full differential ($\mathcal{D}_r \to \partial_r$ **due structure of** W**)**

$$H^{(ch-r)} = Re\ (\partial W/\partial\Phi^i)\partial_r\Phi^i = \partial W/\partial r. \tag{12}$$

Using the Kerr coordinate system $\sqrt{-g} = (r^2 + a^2\cos^2\theta)\sin\theta$, **and axial symmetry we obtain for the mass-energy of the bag**

$$\delta M_{bag} = \int dx^3\sqrt{-g}\ T_0^{\ 0(ch)} = 2\pi\int drd\theta(r^2 + a^2\cos^2\theta)\sin\theta\partial_r W. \tag{13}$$

Superpotential $W(r)$ **is constant inside and outside the source, and on the boundary it has incursion** $\Delta W = W(R+\delta) - W(R-\delta) = -\mu\eta^2$ **. Integration yields**

$$\delta M_{bag} = 2\pi\Delta W\int_{-1}^{1} dX(R^2 + a^2X^2) = 4\pi(R^2 + \frac{1}{3}a^2)\Delta W. \tag{14}$$

FERMIONIC SECTOR DIRAC EQUATION splits in the Weyl representation into two equations

$$\sigma^{\mu}_{\alpha\dot{\alpha}} i\partial_\mu \bar{\chi}^{\dot{\alpha}} = m\phi_\alpha, \quad \bar{\sigma}^{\mu\dot{\alpha}\alpha} i\partial_\mu \phi_\alpha = m\bar{\chi}^{\dot{\alpha}}, \tag{15}$$

the "left-handed" and "right-handed" electron fields, Weyl spinors.

One of them, say "left" field can be associated with spinor structure of the Kerr congruence.

The Kerr theorem determines all the geodesic and *shear free* congruences as analytical solutions of the equation

$$F(T^A) = 0, \tag{16}$$

where F is an arbitrary holomorphic function of the projective twistor variables

$$T^A = \{Y,\ \zeta - Yv,\ u + Y\bar{\zeta}\}, \qquad A = 1, 2, 3, \tag{17}$$

and $\zeta = (x+iy)/\sqrt{2}$, $\quad \bar{\zeta} = (x-iy)/\sqrt{2}$, $\quad u = (z+t)/\sqrt{2}$, $\quad v = (z-t)/\sqrt{2}$ are the null Cartesian coordinates of the Minkowski space $x^\mu = (t, x, y, z) \in M^4$.

Projective spinor coordinate

$$Y = \phi_1/\phi_0, \tag{18}$$

is equivalent to the Weyl two-component spinor ϕ_α.

Two antipodally conjugate solutions of the Kerr theorem $Y^+ = -1/\bar{Y}^-$ determine two Weyl spinor fields ϕ^α and $\bar{\chi}_{\dot{\alpha}}$, corresponding to antipodal congruences $Y^+ = \phi_1/\phi_0 \quad , \quad Y^- = \bar{\chi}^{\dot{1}}/\bar{\chi}^{\dot{0}}$
For Y^+ we have

$$\phi_\alpha = \begin{pmatrix} e^{-i\phi/2}\cos\frac{\theta}{2} \\ e^{i\phi/2}\sin\frac{\theta}{2} \end{pmatrix}, \tag{19}$$

and for $Y^- = -1/\bar{Y}^+$,

$$\bar{\chi}^{\dot{\alpha}} = \begin{pmatrix} -e^{-i\phi/2}\sin\frac{\theta}{2} \\ e^{i\phi/2}\cos\frac{\theta}{2} \end{pmatrix}. \tag{20}$$

Only one of the fields, say "left", $k_\mu^{(+)}(x)$ is "retarded" and corresponds to the external KN solution. The field $k_\mu^{(-)}(x) = (1, -\mathbf{k})$, retains the time-like direction and reflects the space orientation.

The spinor fields created by the Kerr theorem ϕ_α and $\bar{\chi}^{\dot{\alpha}}$ correspond to the left out-field and right-in fields, i.e. the retarded and advanced fields correspondingly. Removing twosheetedness by the bag-source, we meet it again from the external side!

The null vector fields $k^{\mu\pm}(x)$ differ on the retarded and advanced sheets, and generate different metrics

$$g^\pm_{\mu\nu} = \eta_{\mu\nu} + 2H_{(KN)}k^\pm_\mu k^\pm_\nu. \tag{21}$$

The "left" and "right" Weyl components of the Dirac fields should be positioned on SEPARATE SHEETS of the Kerr space-time.
This requirement disappears inside the bag, where the space is flat, and both congruences $k^{\pm}_{\mu}(x)$ are consistent with the flat Minkowski space.

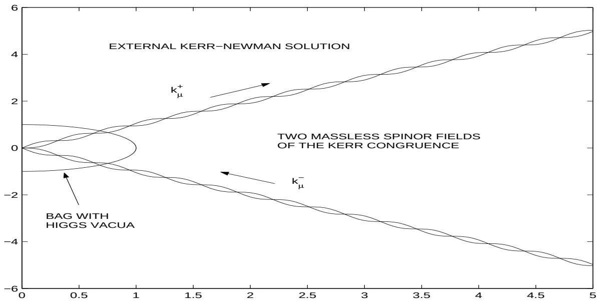


Figure 7: **Spinors ϕ_α $\bar{\chi}^{\dot{\alpha}}$ are placed on different sheets controlled by antipodal Kerr congruences. Inside the bag these spinor fields acquire Yukawa coupling giving mass to the Dirac equation.**

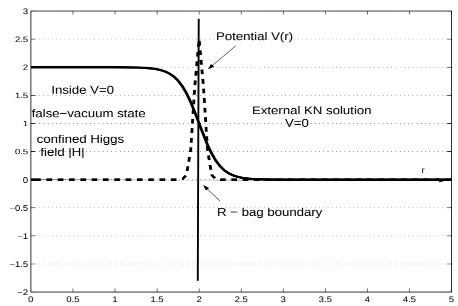


Figure 8: **The KN bag. The Higgs field H forms false-vacuum state inside the bag.**

Extending the left and right spinor fields inside the solitonic bag, we obtain that they transfer into the flat Minkowski space, where the both null congruences turn out to be compatible, so far as these congruences are null with respect to the same flat sheet of the common internal Minkowski space. Inside the soliton they are united into Dirac bispinor $\Psi = \begin{pmatrix} \phi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$, **corresponding to the massless Dirac equation**

$$(\gamma^\mu \partial_\mu)\Psi(x) = 0, \tag{22}$$

and the confined inside the bag Higgs field $\mathcal{H}$ **adds the mass term, generating the Dirac equation with VARIABLE mass term**

$$m \equiv g\mathcal{H}. \tag{23}$$

The variable mass term is a typical feature of the bag models!

The Dirac wave function, solution of the Dirac equation with variable mass term, avoids the region with a large bare mass, and tends to get an energetically favorable position. **In the SLAC bag model [?] this problem is solved by variational approach. The corresponding Hamiltonian is**[1]

$$H(x) = \Psi^\dagger(\frac{1}{i}\vec{\alpha}\cdot\vec{\nabla} + g\beta\sigma)\Psi, \tag{24}$$

and the energetically favorable wave function has to be determined by minimization of the averaged Hamiltonian $\mathcal{H} = \int d^3x H(x)$ **under the normalization condition** $\int d^3x \Psi^\dagger(x)\Psi(x) = 1$. **It yields**

$$(\frac{1}{i}\vec{\alpha}\cdot\vec{\nabla} + g\beta\sigma)\Psi = \mathcal{E}\Psi, \tag{25}$$

where $\mathcal{E}$ **appears as the Lagrangian multiplier enforcing the normalization condition. Similar to results of the SLAC-bag model, one expects that the Dirac wave function will not penetrate deep in the region of large bare mass** $m = g\eta$, **and will concentrate in a narrow transition zone at the bag border** $R - \delta < r < R + \delta$. **As it is explaned in SLAC model, narrow concentration of the Dirac wave function is admissible for scalar potential which does not lead to the Klein paradox.**

[1] Here α and β are the known Dirac matrices.

The exact solutions of this kind are known only for two-dimensional case, and the corresponding variational problem should apparently be solved numerically by using the ansatz $\tilde{\Psi} = f(x)\Psi(x)$, in which $f(x)$ is a variable factor for the Dirac solution based on the Weyl spinors $\phi_\alpha, \bar{\chi}^{\dot{\alpha}}$ consistent with the corresponding outgoing and ingoing Kerr congruences.

The Dirac wave function is strongly deformed. In the conception of the MIT- and SLAC- bag-models, a variational principle is used for obtaining the states of minimal energy.

For the KN bag model, the correlations of the wave function with the Kerr congruence is retained by deformations of the bag.

Solutions are localized in the narrow boundary of bubble – effectively similar to that in the SLAC-bag model.

Stringy deformations of bag: unification of the bare and dressed electron

Taking the bag model conception, we should also accept the dynamical point of view that the bags are soft and may easily be deformed. By deformations the bags may form stringy structures. Usually considered deformations of the bags are radial and rotational excitations, forming the open strings, or flux-tubes – a meson as a string joining the quark-antiquark pair.

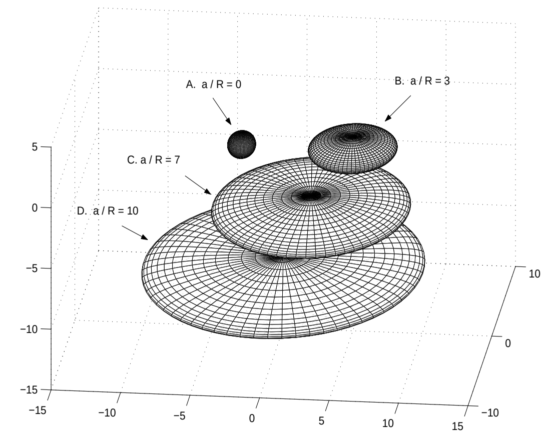


Figure 9: **Non-rotating spherical bad (A), and rotating disk-like bags for different rotational parameters (B):** $a/R = 3$**;(C):** $a/R = 7$**, (D):**$a/R = 10$**.**

The bag-like source of KN solution without rotation, $a = 0$**, represents the Dirac model of a spherical "extensible" electron, which has in rest the classical electron radius** $r_e = e^2/2m$**. The KN bag may be considered as the spherical Dirac bag stretched by rotation to the disk of Compton radius.**

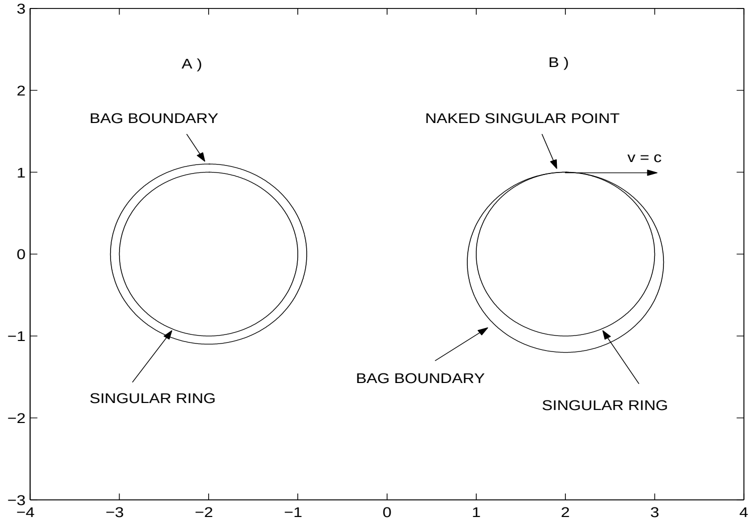


Figure 10: **Regularization of the KN EM field. Section of the disk-like bag in equatorial plane. Distance from positions of the boundary of the bag from position of the (former) singular ring acts as a cut-off parameter R. (A)Axially symmetric KN solution gives a constant cut-off** $R = r_e$. **(B)The boundary of the bag is deformed by a traveling wave, creating a circulating singular point of tangency (zitterbewegung).**

It has been obtained long ago that the Kerr geometry is close related with strings.

In particular, a closed string was associated with the Kerr singular ring. After regularization, the role of this ring-string is played by the sharp boundary of the disk-like bag.

Like the Kerr singular ring, it can serve as carrier of the traveling waves.

It was shown that field structure of this string is similar to structure of the fundamental string, obtained by Sen as a solitonic solution to low energy string theory. The EM field is concentrated on the sharp border of the disk-like source, forming a string-like *frozen* traveling wave with null invariants

$$(\vec{E} \cdot \vec{H}) = 0, \;\; E^2 - H^2 = 0, \quad [E \times H] = \frac{c}{4\pi} \vec{P} \sim \vec{k}. \tag{26}$$

By regularization of the source, the Kerr singular ring-string is regularized with the cut-off parameter R, which for the axially symmetric KN solution is the constant $R = r_e$. The null vector of the Kerr congruence k_μ is tangent to the Kerr singular ring, and since $R << a$ this string is almost light-like, and very close to the known pp-wave strings. However, the light-like KN strings cannot be closed, since the points $x^\mu(\phi, t)$ and $x^\mu(\phi + 2\pi, t)$ do not coincide. There is possibility to consider this string as open one and to complete it to the consistent sum of the left add right modes,

We consider the above "frozen" solution as right mode of excitation and we will complete it by the left counterpart, which can be found among other admissible excitations.

The EM excitations of the Kerr background are defined by analytic function $A = \psi(Y, \tau)/P^2$ where $Y = e^{i\phi} \tan \frac{\theta}{2}$ is a complex projective angular variable, $\tau = t - r - ia \cos\theta$ is a complex retarded-time parameter and $P = 2^{-1/2}(1 + Y\bar{Y})$

for the Kerr geometry at rest. Vector potential is determined by function ψ as follows (DKS)

$$A_\mu dx^\mu = -Re \ [(\frac{\psi}{r + ia\cos\theta})e^3 + \chi d\bar{Y}], \quad \chi = 2\int (1 + Y\bar{Y})^{-2}\psi dY \tag{27}$$

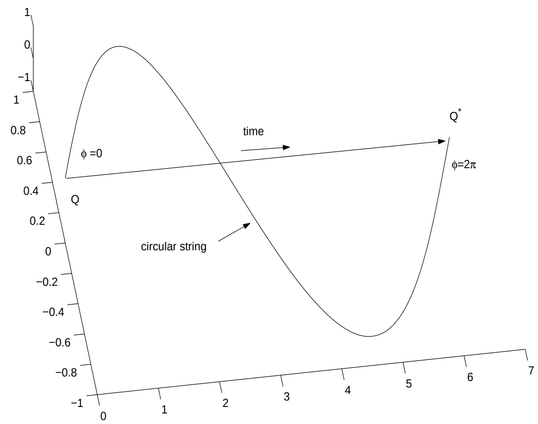


Figure 11: **The circular left mode, formed by traveling wave along the KN string, is completed by the time-like right mode, formed by the frozen traveling wave of the stationary KH solution.**

The simplest function $\psi = -e$ corresponds to stationary KN solution. The constant $\psi = -e$ creates a ***frozen*** circular EM wave, which is locally pp-wave "propagating" along the Kerr singular ring. By regularization, it gets the constant cut-off parameter $R = r_e$.

Along with many other possible stringy waves, interesting effect shows the lowest wave solutions

$$\psi = e(1 + \frac{1}{Y} e^{i\omega\tau}). \tag{28}$$

It is easy to find a back-reaction of this excitation – the corresponding the bag deformation. Boundary of the disk is very close to position of the Kerr singular ring, and regularization of the KN source represents in fact a cut-off parameter $R = r_e$, for the Kerr singularity. The EM traveling waves will deform the bag surface.
Boundary of the deformed bag is determined similarly to the stationary case from the condition $H = 0$.

Function ψ acts on the metric through the function H

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2 \cos^2 \theta} \ , \tag{29}$$

and the condition $H = 0$ determines the boundary of disk $R = |\psi|^2/2m$, which acts as the cut-off parameter for EM field.

One sees that solution

$$\psi = e(1 + \frac{1}{Y} e^{i\omega\tau}) \tag{30}$$

takes in equatorial plane $\cos\theta = 0$ **the form** $\psi = e(1 + e^{-i(\phi - \omega t)})$, **and the cut-off parameter**

$$R = |\psi|^2/2m = \frac{e^2}{m}(1 + \cos(\phi - \omega t))$$

depends on $\phi - \omega t$.

Vanishing R **at** $\phi = \omega t$ **creates singular pole which circulates along the ring-string, reproducing the known zitterbewegung of the Dirac electron. This pole may be interpreted as a single end point of the ring-string, or as a bare point-like electron, either as a light-like quark confined inside the bag.**

The mysterious zitterbewegung of the Dirac electron appears as a manifestation of the ring-string traveling waves.

CONCLUSION:

- **Source of the KN solution represents a supersymmetric, BPS-saturated soliton.**
- **BAG: Consistent implementation of the Dirac equation and emergence the mass from the Yukawa coupling.**
- **BPS-bound determines *unambiguously* shape of the KN bag and its stability,**
- **(e) Unification of the "dressed" and "pointlike" electron.**
- **Stringy structures and formation of the coherent bag-string-quark system:**
- **Consistent space-time implementation gravity in the Standard Model.**
- **Structure of the gravitating KN bag admits extension to other particles of the electroweak sector SM.**

THANK YOU FOR ATTENTION!