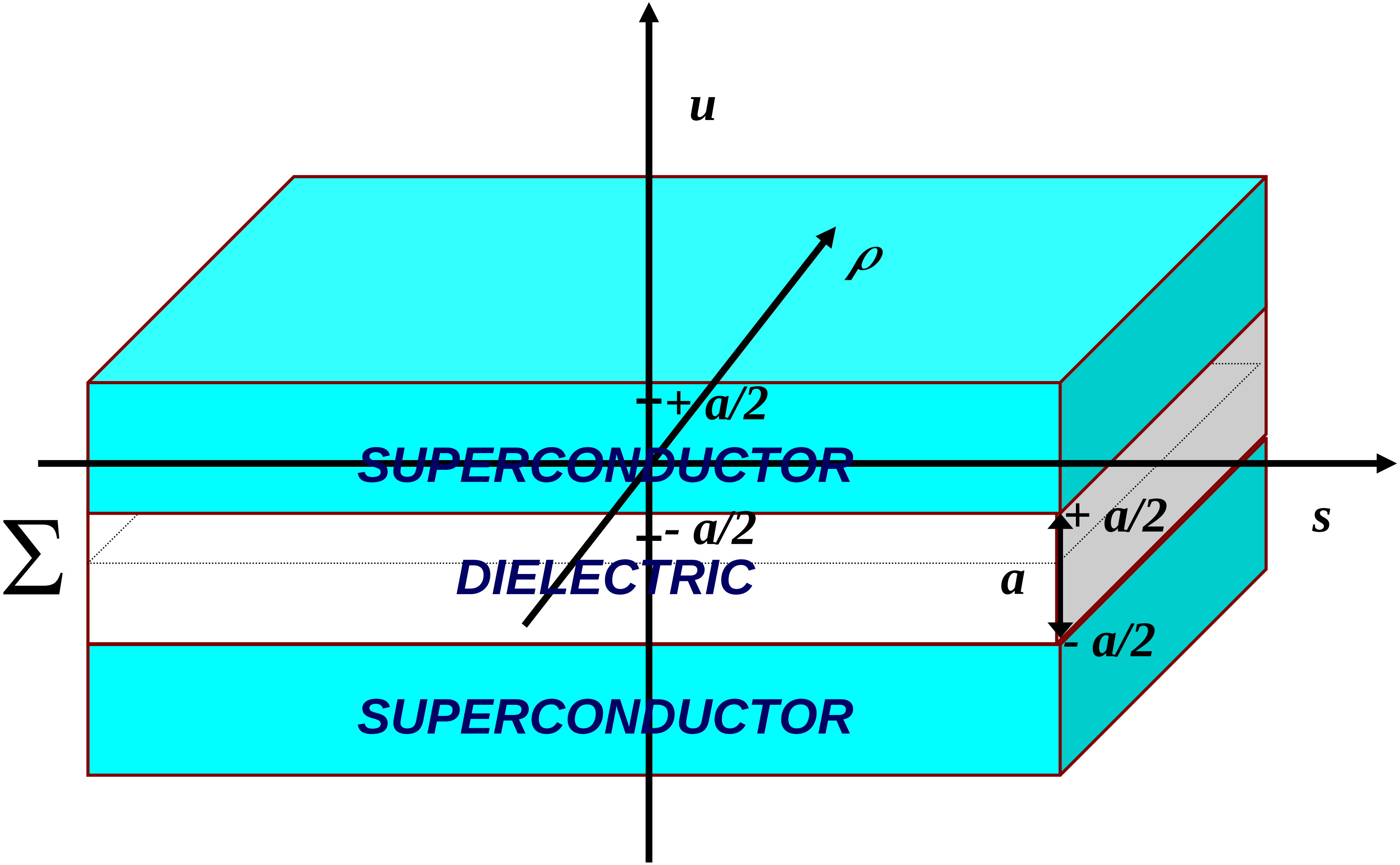
The curvature effects in 1d and 2d Josephson junctions.

Tomasz Dobrowolski

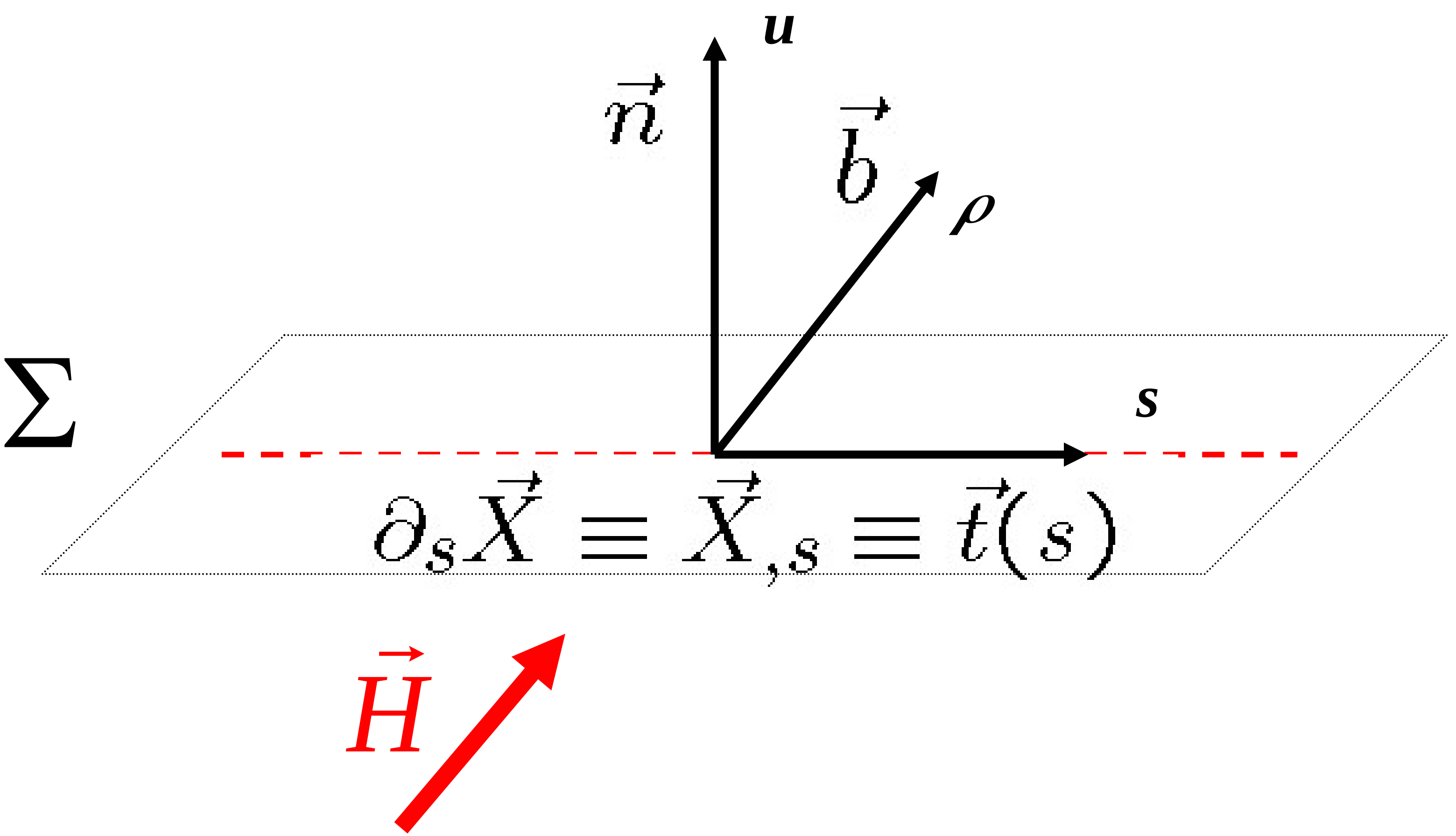
Josephson junction



Dimensions

- Large area Josephson junction (2+1 d)
- Long Josephson junction (1+1 d)
- Point Josephson junction (0+1 d)

1+1d Junction



$$\partial_s \vec{X} \equiv \vec{X}_{,s} \equiv \vec{t}(s)$$

$$\vec{H} = H_\rho \vec{b} \equiv H\,\vec{b}.$$

$$\mathbf{curl}\,\vec{H} = \frac{4\pi}{c}\vec{J} + \frac{\varepsilon}{c}\partial_t\vec{E}$$

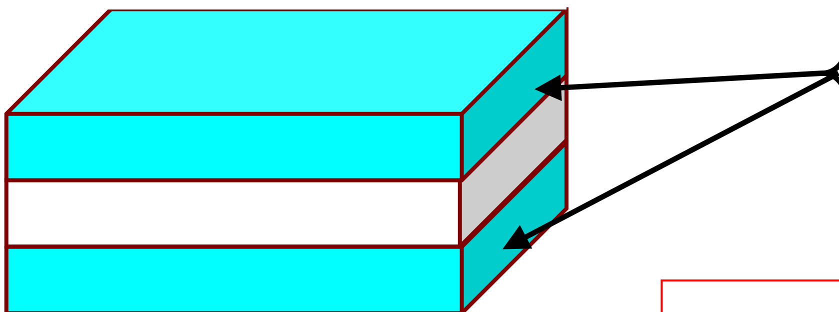
$$\varepsilon\,\mathbf{div}\,\vec{E} = 4\pi\varrho$$

$$\mathbf{curl}\,\vec{E} = -\frac{\mu}{c}\partial_t\vec{H}$$

Maxwell equations

T. Dobrowolski
„Curved Josephson Junction.”
Annals of Physics <u>327</u>, 1336 (2012).

$$\psi_T = |\psi_T| e^{\imath\varphi_T}$$



$$\psi_B = |\psi_B| e^{\imath\varphi_B}$$

$$\begin{cases} \vec{E} = \partial_t\left(\Lambda\vec{J}\right) = \dfrac{4\pi\lambda_L^2}{c^2}\partial_t\vec{J} \\ \vec{J} = \dfrac{q^*}{m^*}\left[\dfrac{1}{2}\imath\,\hbar\left(\psi\nabla\psi^* - \psi^*\nabla\psi\right) - \dfrac{q^*}{c}\vec{A}\psi\psi^*\right] \end{cases}$$

$$\phi(s,t) \equiv \varphi(s,\frac{a}{2},t) - \varphi(s,-\frac{a}{2},t) - \frac{q^*}{\hbar c}\int_{-a/2}^{a/2} du\,A_u =$$

$$= \varphi(s,\frac{a}{2},t) - \varphi(s,-\frac{a}{2},t) = \varphi_T - \varphi_B$$

$$-\frac{1}{\bar{c}^2}\partial_t^2\phi(s,t)+\mathcal{F}\partial_s^2\phi(s,t)=\frac{1}{\lambda_J^2}\sin\phi$$

$$\mathcal{F}=\frac{1}{aK}\ln\left(\frac{2+aK}{2-aK}\right)$$

$$\frac{1}{\lambda_J^2}=\frac{8\pi^2(2\lambda_L+\mu a)}{c\Phi_0}J_m.$$

$$\frac{1}{\bar{c}^2}=\varepsilon_I\left(\frac{2\lambda_L}{a}+1\right)\frac{1}{c^2}$$

$$\left(s\rightarrow\frac{1}{\lambda_J}s,\ t\rightarrow\frac{\bar{c}}{\lambda_J}t\right)$$

$$\partial_t^2\phi(s,t)-\mathcal{F}\partial_s^2\phi(s,t)+\sin\phi=0$$

sine-Gordon model on the plane curve

$$\mathcal{L} = \frac{1}{2}\eta_M^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)$$

$$V(\phi) = 1 - \cos\phi$$

$$x^i \longrightarrow \frac{x^i}{\lambda_J} \qquad t \to \omega_P t$$

$$\mathcal{L} = \frac{1}{2}(\partial_t\phi)^2 - \frac{1}{2}\eta_E^{ij}(\partial_i\phi)(\partial_j\phi) - V(\phi)$$

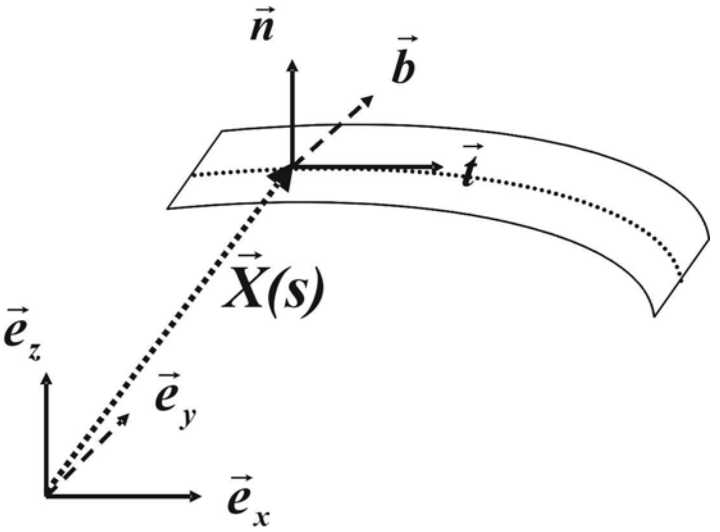
$$\mathcal{L} = \frac{1}{2}(\partial_t\phi)^2 - \frac{1}{2}G^{\alpha\beta}(\partial_\alpha\phi)(\partial_\beta\phi) - V(\phi)$$

$$\xi^\alpha = (\xi^1, \xi^2, \xi^3) = (s, \rho^1, \rho^2) = (s, \rho, u)$$

Connection between curved and Cartesian coordinates

$$\xi^\alpha = (\xi^1, \xi^2, \xi^3) = (s, \rho^1, \rho^2) = (s, \rho, u)$$

$$\vec{x} = \vec{X}(s) + \rho^j \vec{n}_j(s)$$



$$G_{\alpha\beta} = \frac{\partial x^i}{\partial \xi^\alpha} \frac{\partial x^j}{\partial \xi^\beta} \eta^E_{ij}$$

$$G_{ij} = \delta_{ij}, \quad G_{is} = 0, \quad G_{ss} = \mathcal{G}^2 = (1 - uK(s))^2$$

$$G^{ij} = \delta^{ij}, \quad G^{is} = 0 \quad G^{ss} = \frac{1}{G}$$

$$G = \mathcal{G}^2 = (1 - uK)^2$$

$$\mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2G}(\partial_s \phi)^2 - V(\phi)$$

$$L_{eff} = \int ds d\rho du \sqrt{G}\, \mathcal{L}$$

$$L_{eff} = \int ds \mathcal{L}_{eff}$$

$$\int_{-a/2}^{a/2} du(1 - uK) = a$$

$$\int_{-a/2}^{a/2} du \frac{1}{1-uK} = a\mathcal{F}$$

$$\mathcal{L}_{eff} = ab\left(\frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}\mathcal{F}(\partial_s \phi)^2 - V(\phi)\right)$$

$$\mathcal{F} = \frac{1}{aK} \ln\left(\frac{2+aK}{2-aK}\right)$$

$$\partial_t \left(\frac{\delta \mathcal{L}_{eff}}{\delta(\partial_t \phi)}\right) + \partial_s \left(\frac{\delta \mathcal{L}_{eff}}{\delta(\partial_s \phi)}\right) - \frac{\delta \mathcal{L}_{eff}}{\delta \phi} = 0$$

$$\partial_t^2 \phi - \partial_s (\mathcal{F} \partial_s \phi) + \sin\phi = 0$$

T. Dobrowolski

The kink motion in a curved Josephson junction.” **Physical Review E 79, 046601 (2009).**

2+1 d Josephson junction

$$\mathcal{L} = \frac{1}{2}\eta_M^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V(\phi)$$

$$V(\phi) = 1 - \cos\phi$$

$$\eta_M^{\mu\nu} = diag(+1, -1, -1, -1)$$

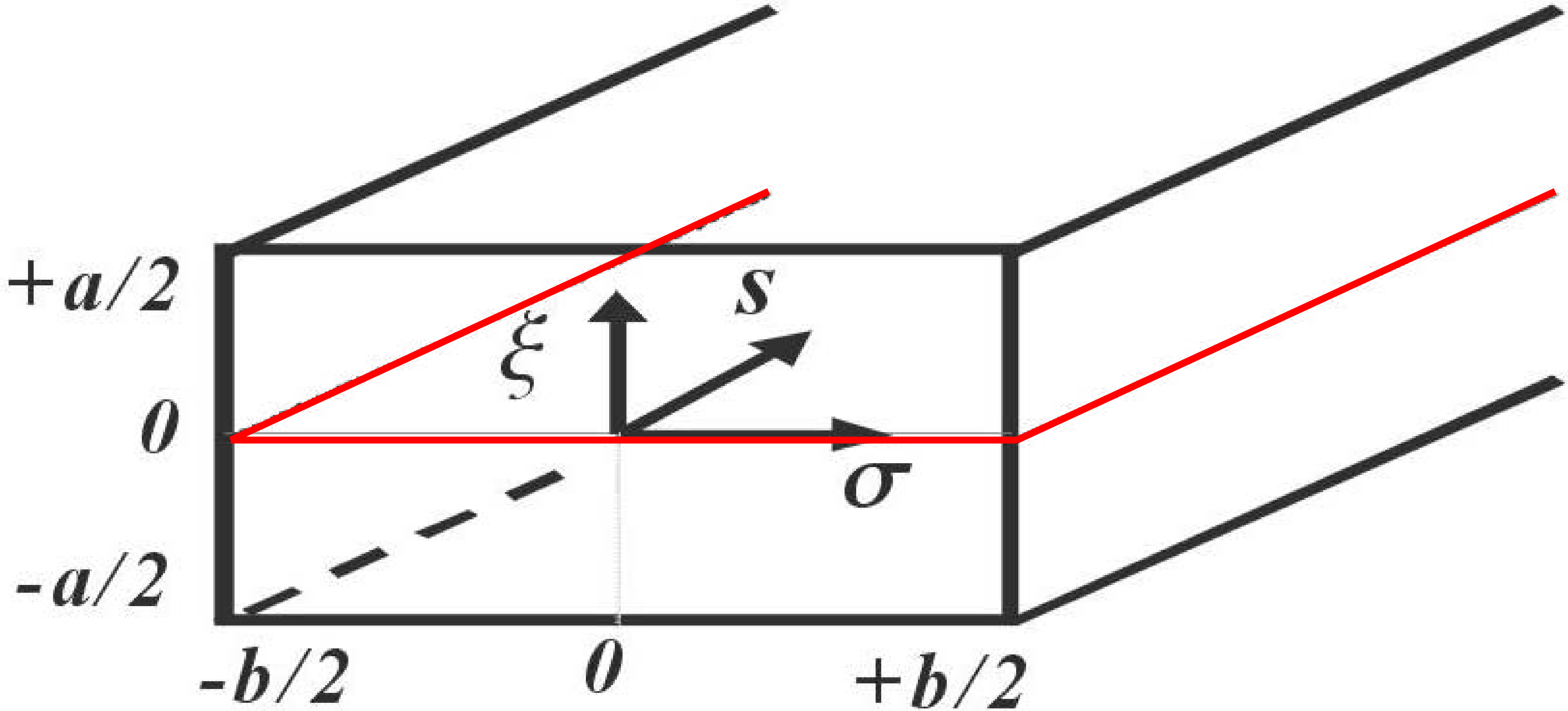
$$t = \omega_P T, \qquad x^i = X^i/\lambda_J$$

ω_P - plasma frequency

λ_J - Josephson length

(T, X^i) - Cartesian coordinates

JOSPHSON JUNCTION - A SUFACE IN 3 DIMENSIONS



$$(\sigma^a) = (\sigma^1, \sigma^2) = (s, \sigma) \qquad \xi^\alpha = (\xi^1, \xi^2, \xi^3) = (\sigma^1, \sigma^2, \xi)$$

Gauss-Weingarten formulas

$$\partial_a \vec{X}_{,b} = \vec{X}_{,ab} = \Gamma^c_{ab} \vec{X}_{,c} - K_{ab} \vec{n} \tag{7}$$

$$\partial_a \vec{n} = \vec{n}_{,a} = K^c_a \vec{X}_{,c} \tag{8}$$

Γ^c_{ab} are a Christoffel symbols calculated with respect to the metric induced on the surface Σ.

$$\boxed{K_{ab} = -\vec{n} \cdot \vec{X}_{,ab}}$$

$\mathcal{R}$ is a Riemann curvature scalar

$$\mathcal{R} = K^a_a K^c_c - K^a_c K^c_a$$

REDUCED MODEL IN 2+1 DIMENSIONS

$$L = \int_{-l/2}^{+l/2} ds \int_{-b/2}^{+b/2} d\sigma \int_{-a/2}^{+a/2} d\xi \sqrt{G} \mathcal{L}$$

ϕ does not depend on ξ.

$$L = \int_{-l/2}^{+l/2} ds \int_{-b/2}^{+b/2} d\sigma \sqrt{g} \mathcal{L}_2$$

where the lagrangian density is defined by

$$\mathcal{L}_2 = \frac{1}{2}\mathcal{C}(\partial_t \phi)^2 - \frac{1}{2}\mathcal{M}^{ab}(\partial_a \phi)(\partial_b \phi) - \mathcal{C}V(\phi)$$

T. Dobrowolski „ The dynamics of the kink in curved large area Josephson Junction."**Discrete and Continuous Dynamical Systems S 4, 1095 (2011).**

$$\mathcal{M}^{ab} = (1 - \tfrac{a^2}{24}\mathcal{R})g^{ab} + \tfrac{a^2}{12}K^{ac}K_c^b$$

$$\mathcal{C} = 1 + \tfrac{a^2}{24}\mathcal{R}$$

Gravitational form

$$L = \int_{-l/2}^{+l/2} ds \int_{-b/2}^{+b/2} d\sigma \sqrt{g}\mathcal{L}_2$$

$$L = \int_{-l/2}^{+l/2} ds \int_{-b/2}^{+b/2} d\sigma \sqrt{\mathrm{g}}\tilde{\mathcal{L}}_2$$

$$\boxed{\sqrt{g}\mathcal{L}_2 = \sqrt{\mathrm{g}}\tilde{\mathcal{L}}_2}$$

$$(\alpha) = (0, a) = (0, 1, 2)$$

$$(\mathrm{g}^{\alpha\beta}) = \begin{pmatrix} \mathrm{g}^{00} & 0 \\ 0 & \mathrm{g}^{ab} \end{pmatrix}$$

Zero order approximation

$$\varepsilon \equiv \frac{a^2}{24}\mathcal{R}$$

$$\mathrm{g}^{00} = 1, \quad \mathrm{g}^{0a} = 0, \quad \mathrm{g}^{ab} = -g^{ab}$$

$$\tilde{\mathcal{L}}_2 = \frac{1}{2}\,\mathrm{g}^{\alpha\beta}(\partial_\alpha\phi)(\partial_\beta\phi) - V(\phi)$$

First order approximation

$$\mathrm{g}^{00} = 1 + \frac{a^2}{20}\left(\mathcal{R} - \frac{1}{3g}K^a{}_b K^b{}_a\right)$$

$$\mathrm{g}^{0a} = 0$$

$$\mathrm{g}^{ab} = -g^{ab} + \frac{a^2}{30}g^{ab}\left(\mathcal{R} + \frac{1}{2g}K^a{}_b K^b{}_a\right) - \frac{a^2}{12}K^{ac}K^b_c$$

$$\tilde{\mathcal{L}}_2 = \frac{1}{2}\,\mathrm{g}^{\alpha\beta}(\partial_\alpha\phi)(\partial_\beta\phi) - \tilde{V}(\phi)$$

$$\tilde{V}(\phi) = V(\phi) + \frac{a^2}{20}\left(\mathcal{R} - \frac{1}{3g}K^a{}_b K^b{}_a\right)V(\phi)$$

Orders of magnitude

- **Gravity**

$$\varepsilon = \frac{R_s}{r}, \qquad R_s = \frac{2G_3M}{c^2}$$

$R_{sSun} \approx 3\text{km}, \quad r_{Sun} \approx 7 \cdot 10^5\text{km}$

$\rightarrow \underline{\varepsilon \sim 0.5 \cdot 10^{-5}}$

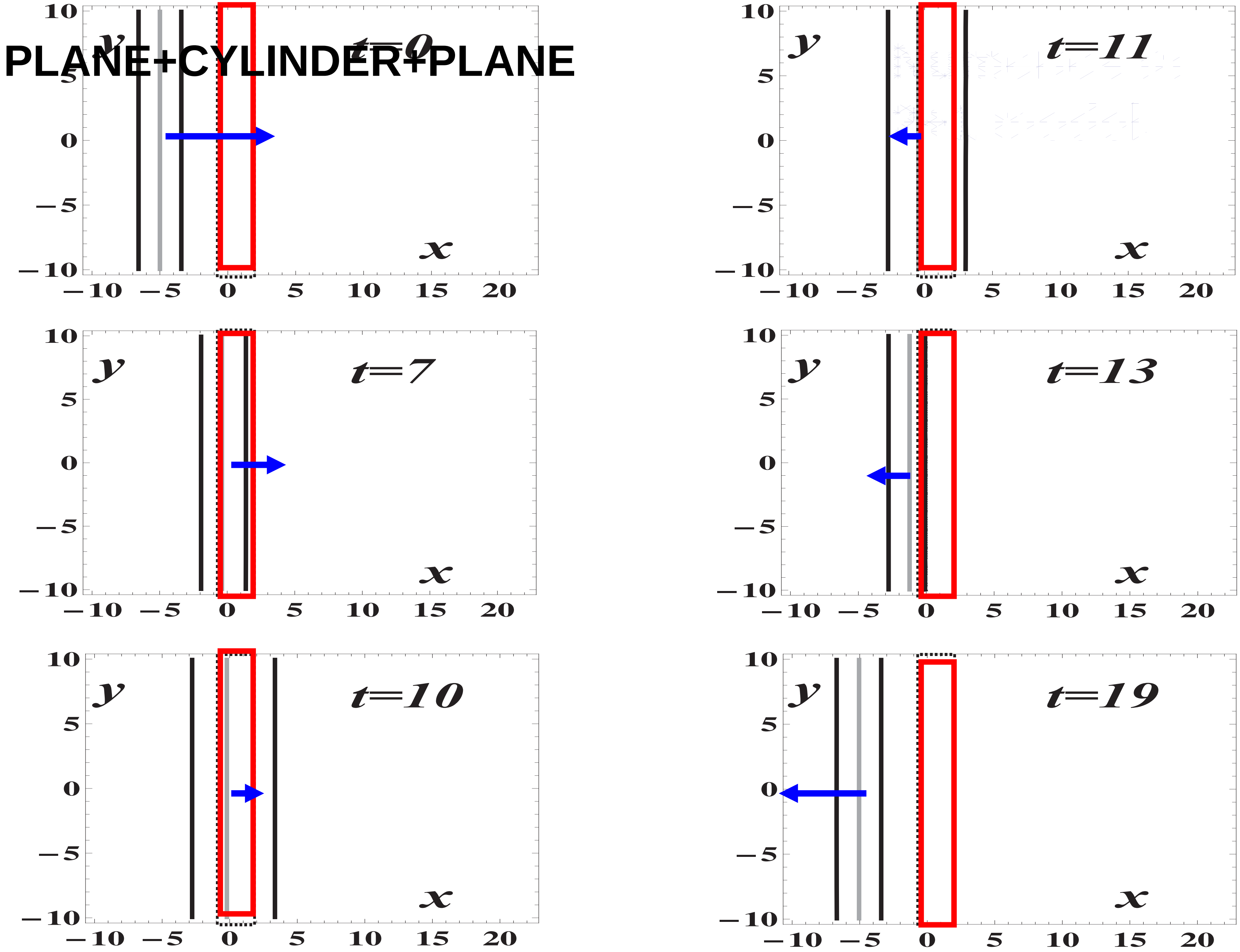
$R_{sEarth} \approx 1\text{cm}, \quad r_{Earth} \approx 6.4 \cdot 10^8\text{cm}$

$\rightarrow \underline{\varepsilon \sim 10^{-9}}$

- **Josephson Junction**

$$\varepsilon \equiv \frac{a^2}{24}\mathcal{R} \qquad a \sim 10^{-2}\lambda_J \qquad \varepsilon \sim 10^{-5}\lambda_J^2\mathcal{R} \qquad \underline{\varepsilon \sim 10^{-5}}$$

$$r_{min} = a/2 \rightarrow \mathcal{R} \sim 4/a^2 \rightarrow \underline{\varepsilon \sim 1/6}$$



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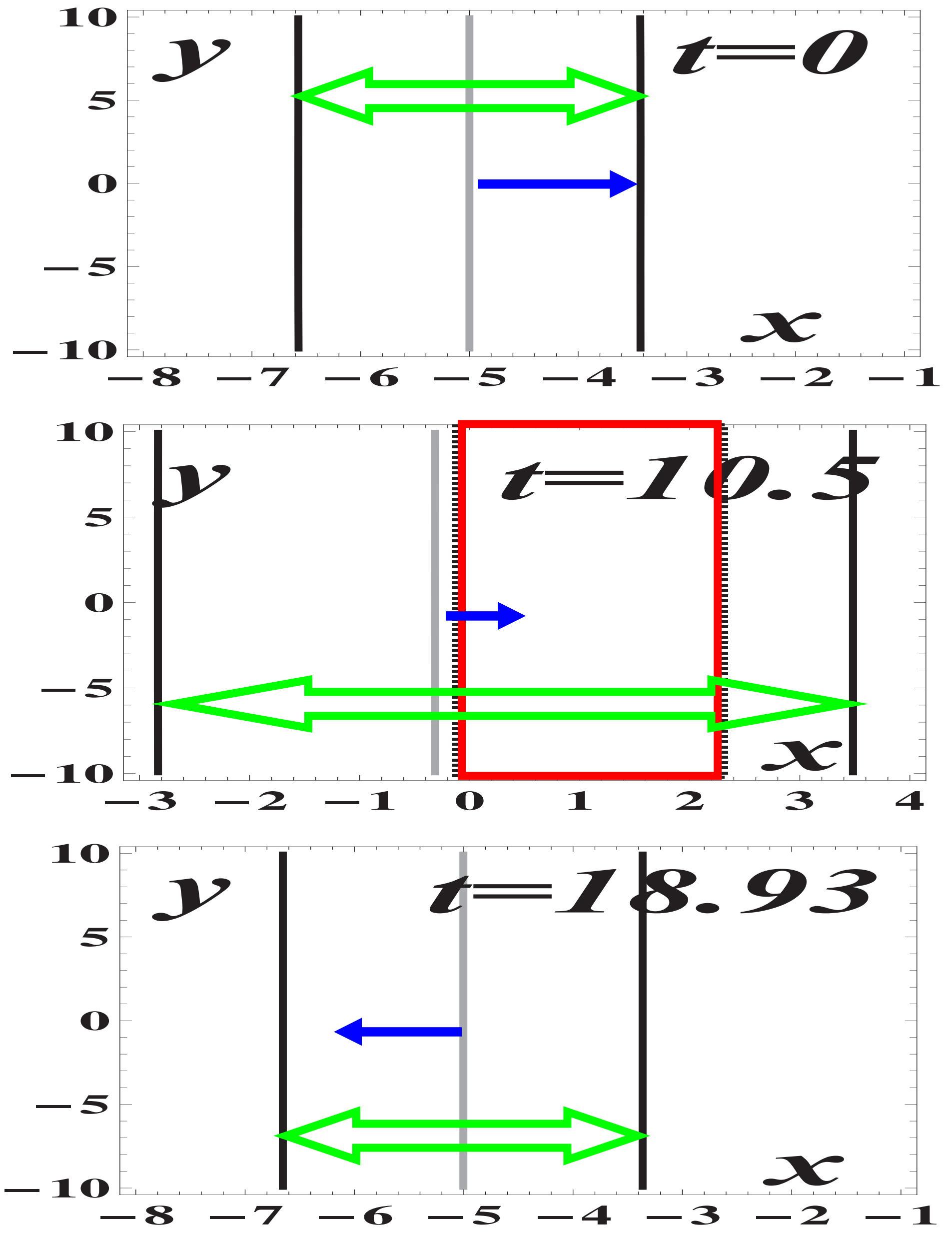
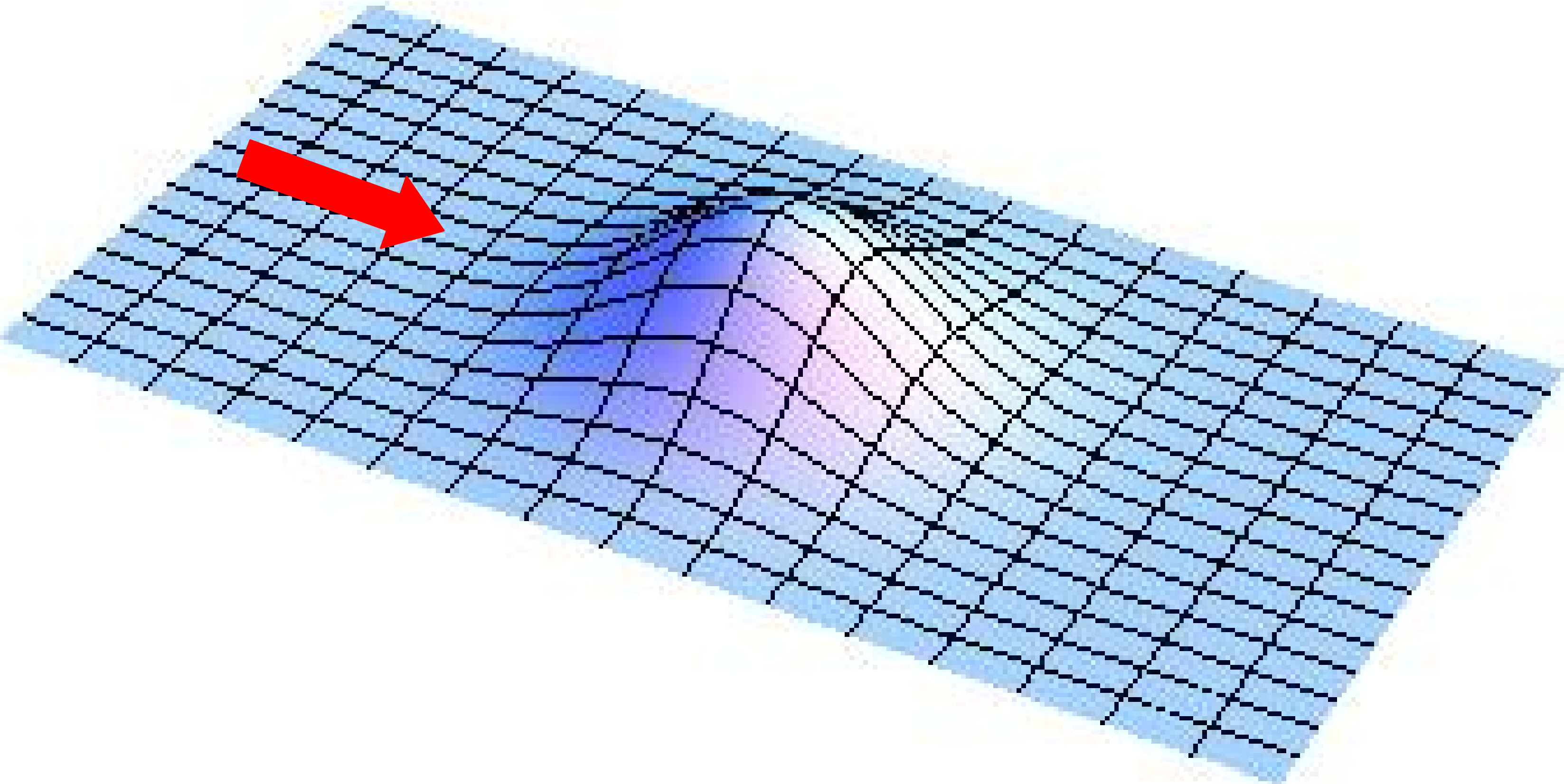
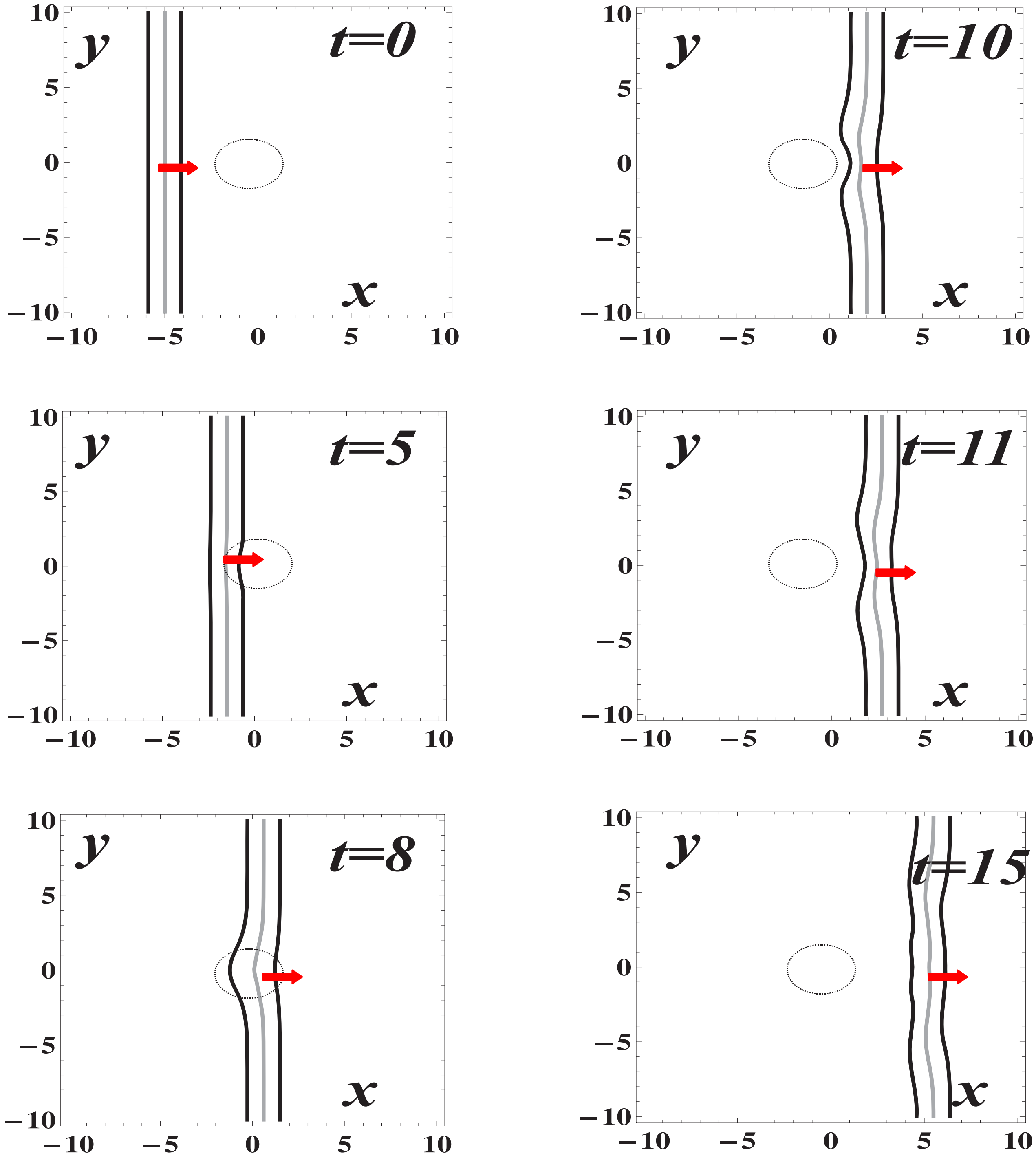
t=11
t=13
t=19
y
x

PLANE+CYLINDER +PLANE



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t=11
t=15
y
x
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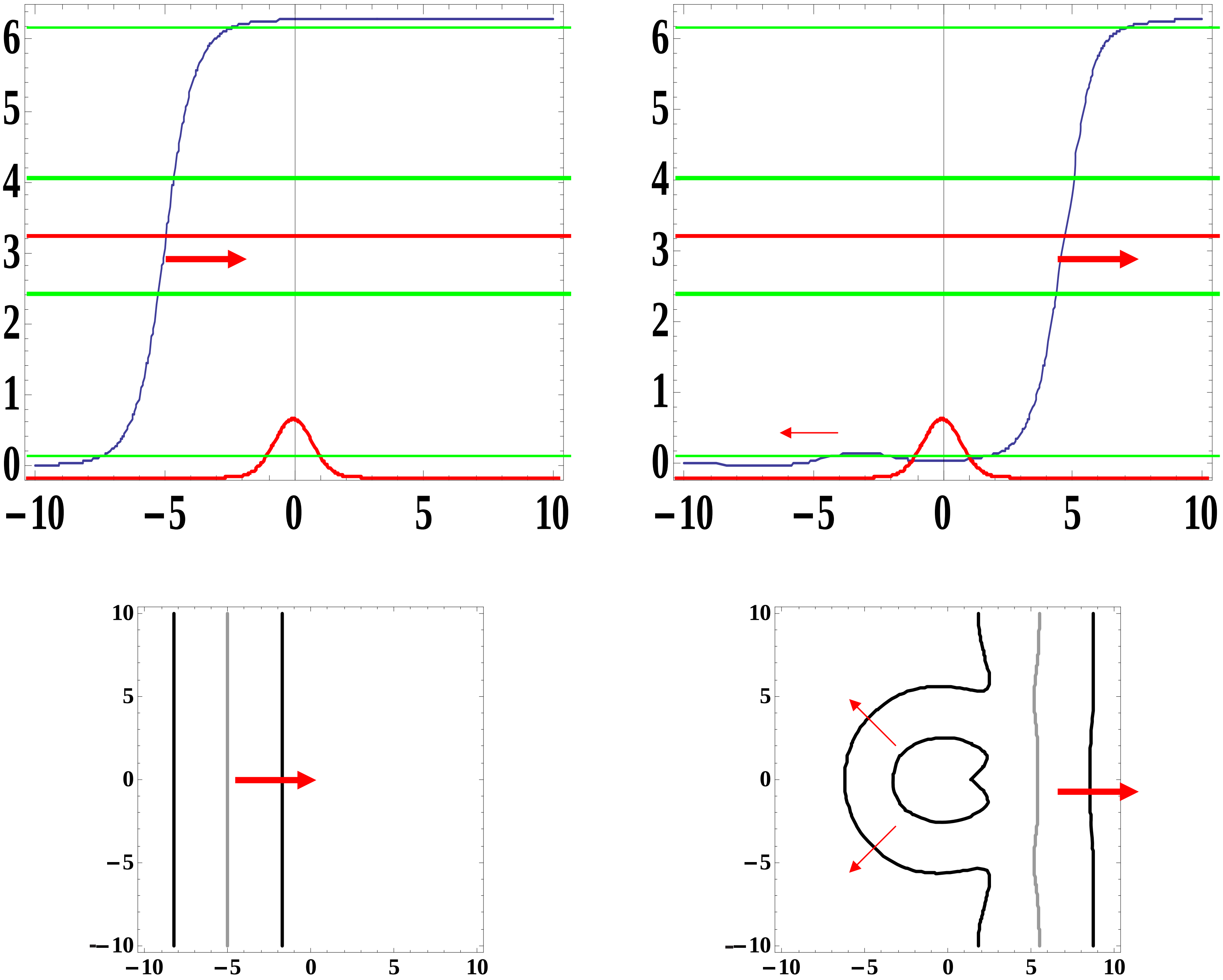
0
−5
−10



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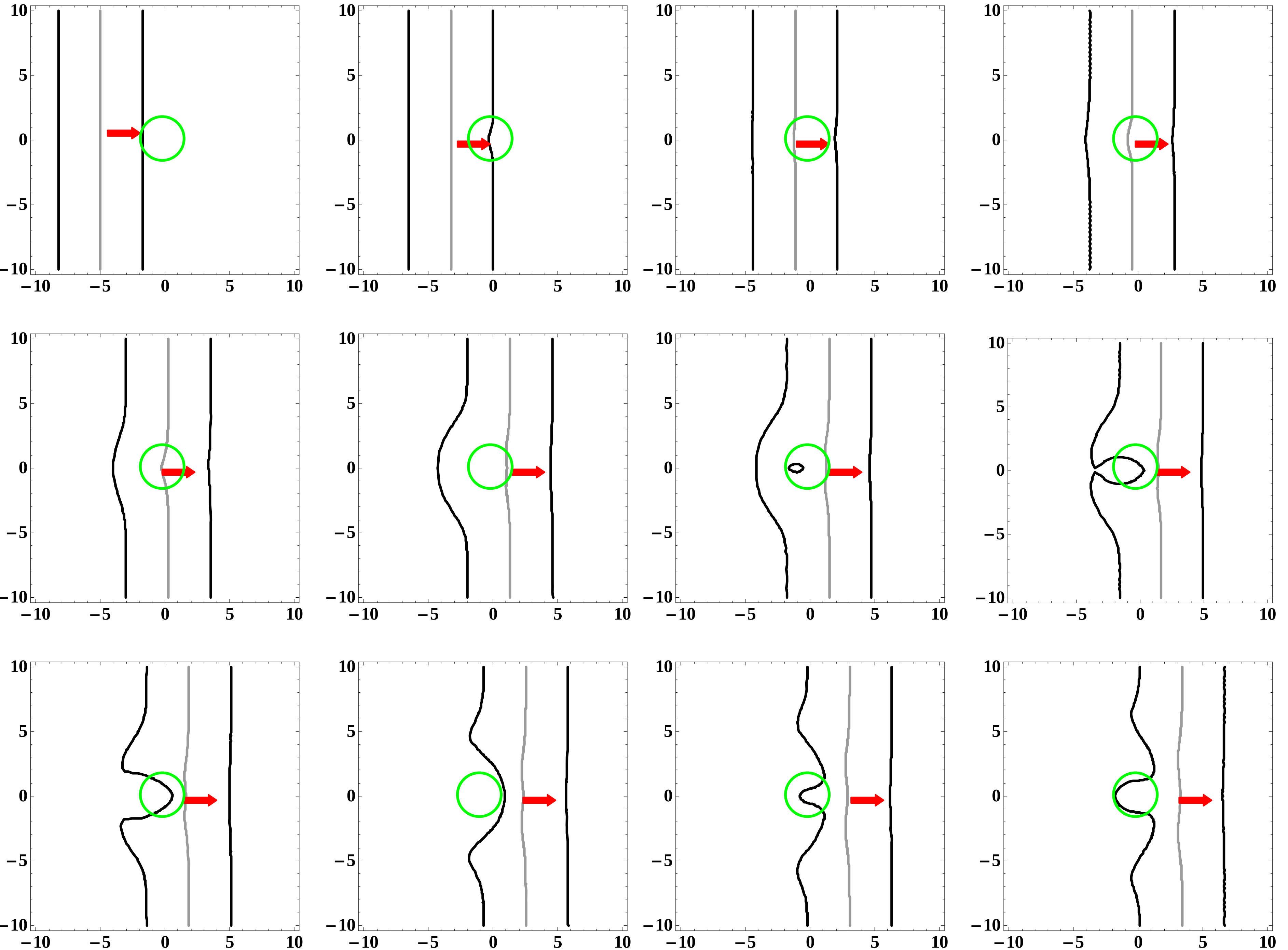
5
4
3
2
1
0
−10
−5
0
5
10
−10
−5
0
5
10



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5
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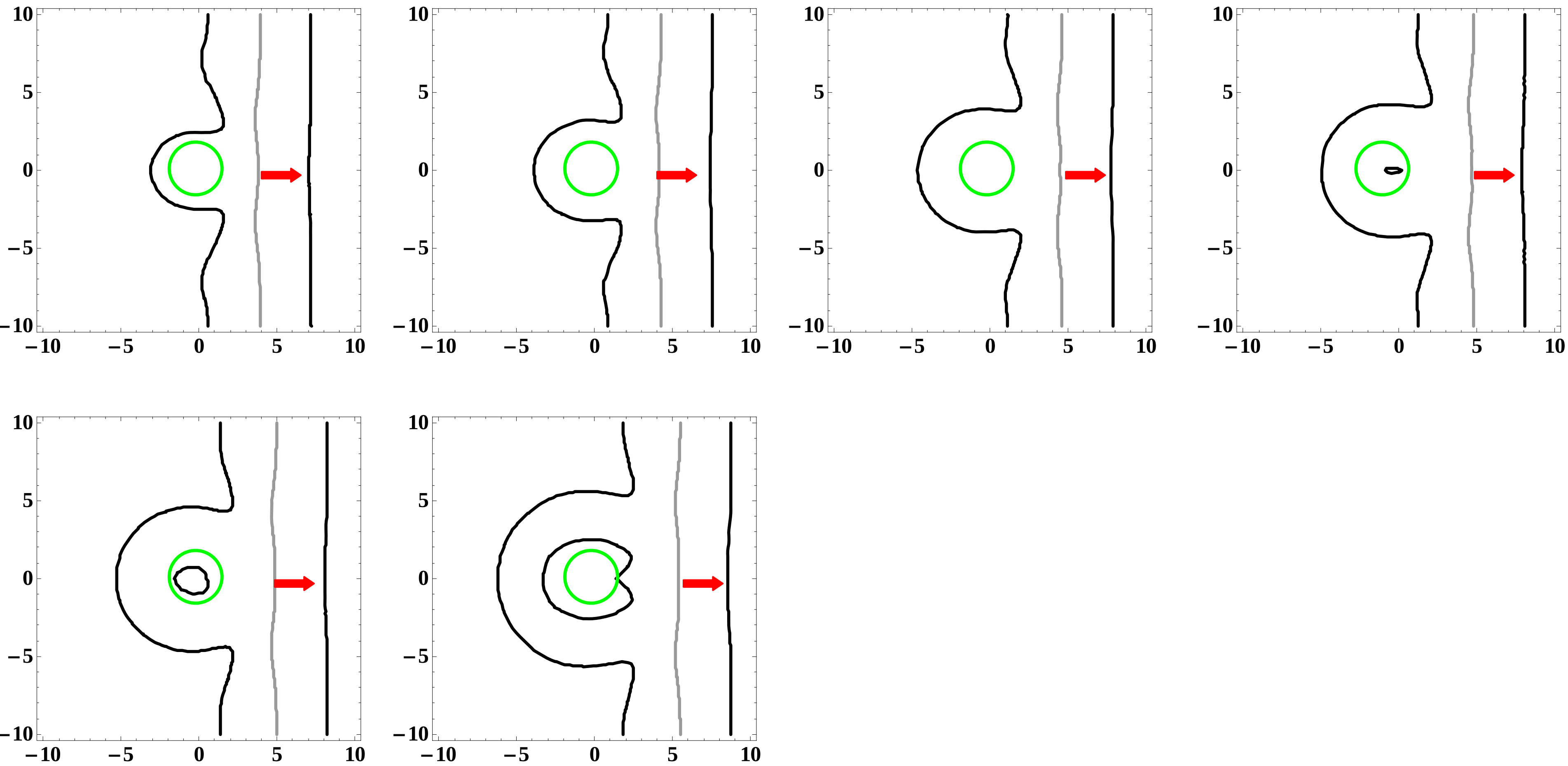
−5
−10
−10
−5
0
5
10


10
5
0
−5
−10
−10
−5
0
5
10

CONCLUSIONS

- THE JUNCTION CAN MIMICS GRAVITY ONLY FOR WEEK FIELDS
- KINK FRONT MOVES SLOWLY IN CURVED REGIONS OF THE JUNCTION (THE SHAPE OF THE POTENTIAL BARIER IS STRICTLY CORRELATED WITH THE GEOMETRY OF THE JUNCTION)
- THE WIDTH OF THE KINK IN CURVED REGIONS GROVES
- DURING THE INTERACTION OF THE KINK FRONT WITH THE CURVED REGIONS OF THE JUNCTION ONE CAN OBSERVE CREATION OF THE WAVES