QUARKS FROM THE TANGENT BUNDLE

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Palette of Quarks

	Color 1	Color 2	Color 3
$t = u^1$	$-\varepsilon^+ \boldsymbol{I}'^+_{23}$	$\varepsilon^+ \boldsymbol{I}^+_{23} \boldsymbol{P}^+_2$	$\varepsilon^+ \boldsymbol{I}^+_{23} \boldsymbol{P}^-_2$
$b = d^1$	$-\varepsilon^+ \boldsymbol{I}'^-_{23}$	$\varepsilon^+ \boldsymbol{I}^-_{23} \boldsymbol{P}^+_3$	$\varepsilon^+ \boldsymbol{I}^-_{23} \boldsymbol{P}^-_3$
$\bar{b} = \bar{d}^1$	$-\varepsilon^- \boldsymbol{I}'^+_{23}$	$\varepsilon^- \boldsymbol{I}^+_{23} \boldsymbol{P}^-_3$	$\varepsilon^- \boldsymbol{I}^+_{23} \boldsymbol{P}^+_3$
$\bar{t} = \bar{u}^1$	$-\varepsilon^- \boldsymbol{I}'^-_{23}$	$\varepsilon^- \boldsymbol{I}^-_{23} \boldsymbol{P}^-_2$	$\varepsilon^- \boldsymbol{I}^-_{23} \boldsymbol{P}^+_2$
$c = u^2$	$\varepsilon^+ \boldsymbol{I}^+_{31} \boldsymbol{P}^-_3$	$-\varepsilon^+ \boldsymbol{I}'^+_{31}$	$\varepsilon^+ \boldsymbol{I}^+_{31} \boldsymbol{P}^+_3$
$s = d^2$	$\varepsilon^+ \boldsymbol{I}^-_{31} \boldsymbol{P}^-_1$	$-\varepsilon^+ \boldsymbol{I}'^-_{31}$	$\varepsilon^+ \boldsymbol{I}^-_{31} \boldsymbol{P}^+_1$
$\bar{s} = \bar{d}^2$	$\varepsilon^- \boldsymbol{I}^+_{31} \boldsymbol{P}^+_1$	$-\varepsilon^- \boldsymbol{I}'^+_{31}$	$\varepsilon^- \boldsymbol{I}^+_{31} \boldsymbol{P}^-_1$
$\bar{c} = \bar{u}^2$	$\varepsilon^- \boldsymbol{I}^-_{31} \boldsymbol{P}^+_3$	$-\varepsilon^- \boldsymbol{I}'^-_{31}$	$\varepsilon^- \boldsymbol{I}^-_{31} \boldsymbol{P}^-_3$
$u = u^3$	$\varepsilon^+ \boldsymbol{I}^+_{12} \boldsymbol{P}^+_1$	$\varepsilon^+ \boldsymbol{I}^+_{12} \boldsymbol{P}^-_1$	$-\varepsilon^+ \boldsymbol{I}'^+_{12}$
$d = d^3$	$\varepsilon^+ \boldsymbol{I}^-_{12} \boldsymbol{P}^+_2$	$\varepsilon^+ \boldsymbol{I}^-_{12} \boldsymbol{P}^-_2$	$-\varepsilon^+ \boldsymbol{I}'^-_{12}$
$\bar{d} = \bar{d}^3$	$\varepsilon^- \boldsymbol{I}^+_{12} \boldsymbol{P}^-_2$	$\varepsilon^- \boldsymbol{I}^+_{12} \boldsymbol{P}^+_2$	$-\varepsilon^- \boldsymbol{I}'^+_{12}$
$\bar{u} = \bar{\bar{u}}^3$	$\varepsilon^- \boldsymbol{I}^-_{12} \boldsymbol{P}^-_1$	$\varepsilon^- \boldsymbol{I}^-_{12} \boldsymbol{P}^+_1$	$-\varepsilon^- \boldsymbol{I}'^-_{12}$

2. CARTAN

- Fix the point, move the frame

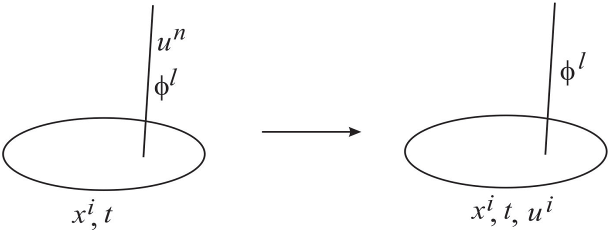
$$dx^i + \omega^i + x^j \omega_j^i = 0$$

- Differentiate and substitute dx^j

$$(d\omega^i - \omega^j \wedge \omega_j^i) + x^j (d\omega_j^i - \omega_j^k \wedge \omega_k^i) = 0$$
$$d\omega^i - \omega^j \wedge \omega_j^i = 0$$
$$d\omega_i^j - \omega_i^k \wedge \omega_k^j = 0$$

- In general, active and passive transformations are not equivalent
- Hence, point is absent

3. Refibrate in Finsler bundle in order to understand



$$\boldsymbol{u} \to \boldsymbol{e}_0$$

$$0 = d\boldsymbol{u} = d\boldsymbol{e}_0 = \omega_0^i \boldsymbol{e}_i$$

Hence,

$$\omega_0^i = 0, \;\; dx^i - u^i dt = 0$$

I am simply illustrating that particles are represented by the origins of moving frames

4. Solving the problem of representing particles, with implications for QM

- Dimension 5
- Fifth dimension τ. On curves: propertime
- Translation element:

$$d\mathscr{P} = \omega^{\mu} \boldsymbol{e}_{\mu} + d\tau \boldsymbol{u},$$

$$\boldsymbol{e}_A \cdot \boldsymbol{e}_B = \eta_{Ab} = diagonal(-1, 1, 1, 1, 1)$$

- Metric

$$d\mathscr{P}(\cdot,\cdot)d\mathscr{P} = 0$$

$$-dt \cdot dt + \sum_i \omega^i \cdot \omega^i + d\tau \cdot d\tau + 2\omega^{\mu} \cdot d\tau = 0$$

$$\omega^{\mu} \cdot d\tau = 0 \Rightarrow d\tau \cdot d\tau = dt \cdot dt - \sum(\omega^i \cdot \omega^i)^2$$

Hence $\omega^{\mu} \cdot d\tau = 0 \Rightarrow d\tau \cdot d\tau = dt \cdot dt - \sum(\omega^i \cdot \omega^i)^2$
But $\omega^{\mu} \cdot d\tau = 0$ is inconsistent with metric
since, on curves, all differential forms are multiples of just one.

5. EXTERIOR ALGEBRA

$$\boldsymbol{a}_i \wedge \boldsymbol{a}_j \quad versus \quad \boldsymbol{a}_i \times \boldsymbol{a}_j,$$

$$\boldsymbol{u} \wedge \boldsymbol{v} = (u^1 v^2 - u^2 v^1)\boldsymbol{a}_1 \wedge \boldsymbol{a}_2 + (\ldots)\boldsymbol{a}_2 \wedge \boldsymbol{a}_3 + \ldots$$

$$\boldsymbol{a}_i \wedge \boldsymbol{a}_j = -\boldsymbol{a}_j \wedge \boldsymbol{a}_i$$

$$dx^\mu \wedge dx^\nu = -dx^\nu \wedge dx^\mu$$

TANGENT CLIFFORD ALGEBRA

- $\boldsymbol{uv} = \frac{1}{2}(\boldsymbol{uv} + \boldsymbol{vu}) + \frac{1}{2}(\boldsymbol{uv} - \boldsymbol{vu})$
- Take $\frac{1}{2}(\boldsymbol{uv} + \boldsymbol{vu})$ to be $\boldsymbol{u} \cdot \boldsymbol{v}$
- There are other choices
- Then (double notation): $\boldsymbol{uv} = \boldsymbol{u} \cdot \boldsymbol{v} + \boldsymbol{u} \wedge \boldsymbol{v} = \boldsymbol{u} \vee \boldsymbol{v}$
- In orthonormal basis $\boldsymbol{e}_\mu \vee \boldsymbol{e}_\nu = \boldsymbol{e}_\mu \wedge \boldsymbol{e}_\nu$ if $\mu \neq \nu$
  $\boldsymbol{e}_\mu \cdot \boldsymbol{e}_\nu = \pm 1, \quad \eta = -1, 1, 1, 1$
  $\gamma_1 \gamma_2 \gamma_3 \gamma_0 \gamma_2 \gamma_1 \gamma_3 = \gamma_1 \gamma_3 \gamma_0 \gamma_1 \gamma_3 = \gamma_3 \gamma_0 \gamma_3 = -\gamma_0$

6. KÄHLER'S ALGEBRA AND CALCULUS

- Kähler algebra is the algebra defined by the relation $dx^\mu dx^\nu + dx^\nu dx^\mu = 2g^{\mu\nu}$
- In pseudo-Cartesian coordinates this is $dx^\mu \cdot dx^\nu = \eta^{\mu\nu}$
- Define (again Cartesian) $d_\mu u = \frac{\partial u}{\partial x^\mu}$
- $\partial u = dx^\mu \vee d_\mu u = du + \delta u$;

  $du = dx^\mu \wedge d_\mu u$; $\delta u = dx^\mu \cdot d_\mu u$;

  δu is co-derivative

  Kähler-Dirac equation: $\partial u = (m + eA)u$

7. Idempotents of Scalar-Valuedness

- The algebra just introduced is of *scalar valued* differential forms.
- Idempotents are essential for solutions with symmetry
- Idempotents of special importance are:

$$\varepsilon^{\pm} = \frac{1}{2}(1 \mp idt); \quad \tau^{\pm} = \frac{1}{2}(1 \pm idx\,dy)$$

- $\varepsilon^{+}\varepsilon^{-} = 0$; $\tau^{+}\tau^{-} = 0$; $\varepsilon^{\pm}\tau^{*} = \tau^{\pm}\varepsilon^{*}$
- $1 = \varepsilon^{+}\tau^{+} + \varepsilon^{+}\tau^{-} + \varepsilon^{-}\tau^{+} + \varepsilon^{-}\tau^{-}$
  Hence

$$u = {}^{+}u^{+}\varepsilon^{+}\tau^{+} + {}^{-}u^{+}\varepsilon^{+}\tau^{-} + {}^{+}u^{-}\varepsilon^{-}\tau^{+} + {}^{-}u^{-}\varepsilon^{-}\tau^{-}$$

$$u = u\varepsilon^{+}\tau^{+} + u\varepsilon^{+}\tau^{-} + u\varepsilon^{-}\tau^{+} + u\varepsilon^{-}\tau^{-}$$

  commutativity allows for annulment:
  $u\varepsilon^{+}\tau^{+} = u^{+}(\varepsilon^{+}\tau^{+}) + 0 + 0 + 0 = u^{+}\varepsilon^{+}\tau^{+}$.

8. Constant Idempotents

- An idempotent c is said to be a constant one iff $d_\mu c = 0$.
- Important property $\partial(u \wedge c) = (\partial u) \wedge c$
- $\partial u = (m + eA)u \Rightarrow \partial(uc) = (m + eA)(uc)$
- The $\varepsilon^{\pm}\tau^{*}$ are constant idempotents. Hence

$$\partial(u\varepsilon^{\pm}\tau^{*}) = (m + eA)(u\varepsilon^{\pm}\tau^{*})$$

  if u is a solution itself
- Hence solutions u are sums of solutions

$$u = u\varepsilon^{+}\tau^{+} + u\varepsilon^{+}\tau^{-} + \ldots + \ldots$$

- But those solutions are not leptons unless they are of the form

$$p(\rho, z, d\rho, dz)e^{i(m\phi - Et)}\varepsilon^{\pm}\tau^{*}.$$

9. Geometrization of the Unit Imaginary

- Rotations: $A \in a_n \to e^{-\frac{1}{2}\phi \boldsymbol{a}_1\boldsymbol{a}_2} A e^{\frac{1}{2}\phi \boldsymbol{a}_1\boldsymbol{a}_2}$, $i \to \boldsymbol{a}_1\boldsymbol{a}_2$; $\quad \boldsymbol{a}_1\boldsymbol{a}_2 = -1$
- By the same token, with $(\boldsymbol{a}_1\boldsymbol{a}_2)^2 = -1$,

$$\frac{1}{2}(1 \pm i dx dy) \to \frac{1}{2}(1 \pm dx dy \boldsymbol{a}_1\boldsymbol{a}_2),$$
$$[\frac{1}{2}(1 \mp i dt)] \to \frac{1}{2}(1 \mp \boldsymbol{a}_0 dt)$$

  we now add
- We have reencountered the double algebra

$$\frac{1}{2}(1 \pm i dy dz) \to \frac{1}{2}(1 \pm dy dz \boldsymbol{a}_2\boldsymbol{a}_3)$$
$$\frac{1}{2}(1 \pm i dz dx) \to \frac{1}{2}(1 \pm dz dx \boldsymbol{a}_2\boldsymbol{a}_3)$$
$$\frac{1}{2}(1 \pm i dx^h) \to \frac{1}{2}(1 \pm \boldsymbol{a}_h dx^h)$$

- But, actually, $\frac{1}{2}(1 \mp i dt) \to \frac{1}{2}(1 \mp \boldsymbol{u} d\tau)$ with $\boldsymbol{u}^2 = -1$.