Neutron star oscillations in alternative theories of gravity

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Plan of the talk:

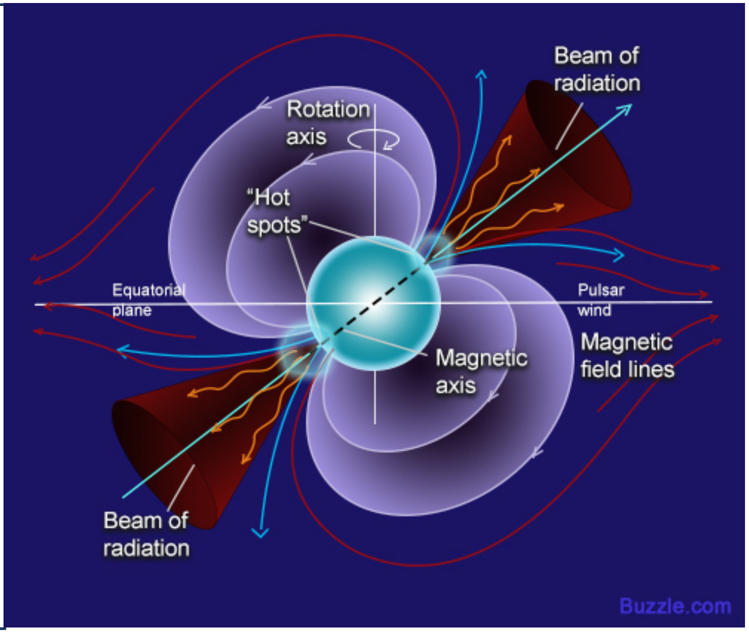
- Neutron stars
- Alternative theories of gravity
- Equilibrium rotating neutron-star solutions
- Oscillations and gravitational wave emission

Neutron stars

A perfect laboratory for testing the strong field regime of gravity

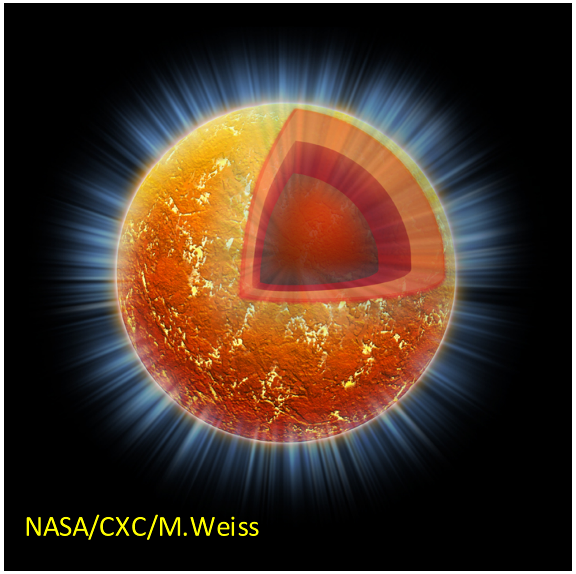
- They are the **most compact stars** known to exist in the universe.
- They have **densities equal to that of the early universe** and **gravity similar to that of a black hole.**
- Most extreme magnetic fields known in the universe up to **10^{16} G**.

- **Conjectured - 1931**
- **Discovered - 1967**
- **Known - 2500+**
- **Mass - 1.2–2$M_{\odot}$**
- **Radius - 8-14 km**
- **Density - 10^{15}g/cm^3**
- **Spin - < 716 Hz**
- **In our Galaxy ~10^8**

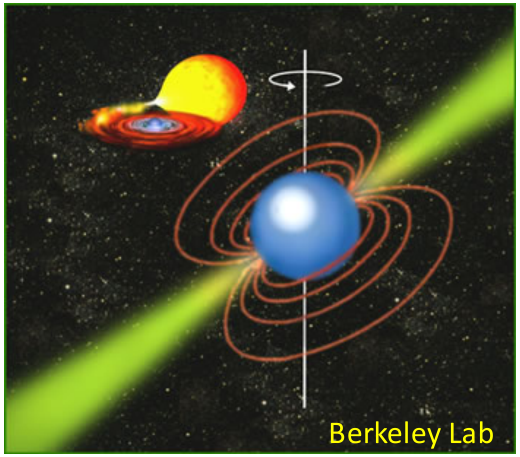


Neutron Stars - Physics in its extremes

- **Strong field effects** of gravity – non-negligible
- **Strong interactions** more important than in any other part of the present universe



➢ **Very large set of observable phenomena**



Neutron Stars - a unique interplay among

- **Astrophysics**
- **Gravitational physics**
- **Nuclear physics**

After half a century since their discovery, we are still far from understanding the composition of matter in their cores!

Alternative Theories of Gravity

Motivation and Overview

Alternative theories of gravity: Motivation

Motivation for modifying General Relativity

Theory

- Theories trying to unify all the interactions: Kaluza-Klein theories, higher dimensional gravity, etc.
- Quantum corrections in the strong field regime
- Studying alternative theories of gravity can give us a deeper insight in GR itself

Observations

- Dark energy and dark matter does not fit well in the standard GR framework
- The strong field regime of gravity is essentially unconstrained

Alternative theories of gravity: Motivation

- There is a very wide range of alternative theories of gravity constructed from different generalizations/modifications of Einstein's theory.
- We will concentrate on the most natural and widely used generalizations:
    - Scalar-tensor theories of gravity
    - $f(R)$ theories of gravity
- They are in agreement with all the observations and do not posses any intrinsic problems.
- Widely used as an alternative explanation of the dark energy phenomena.
- Scalar-tensor theories can be consider as an Einstein theory of gravity but with variable gravitational constant.

Scalar-tensor theories

- **Essence:** one or several scalar fields that can be viewed as mediators of the gravitational interaction in addition to the spacetime metric

- **Action:**

Jordan frame
Physical one

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[F(\Phi)\tilde{R} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)\right] + S_m\left[\Psi_m;\tilde{g}_{\mu\nu}\right]$$

✓ Conformal transformation of the metric
✓ Redefinition of the scalar field

Einstein frame
Much simpler!

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right) + S_m[\Psi_m; A^2(\varphi)g_{\mu\nu}]$$

The price we pay for simplicity: Explicit coupling between the matter and the scalar field

$f(R)$ theories

- **Motivation:** widely used as un alternative explanation of the accelerated expansion of the universe
- **Studied mainly at cosmological scales**, but every theory of gravity should pass via the observations at astrophysical scale too
- **Action:** $S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + S_{\text{matter}}(g_{\mu\nu},\chi)$
- Free of tachyonic instabilities and the appearance of ghosts when:

$$\frac{d^2 f}{dR^2} \geq 0, \qquad \frac{df}{dR} > 0$$

- **Mathematical treatment** of the problem: $f(R)$ theories are mathematically equivalent to a particular class of massive scalar-tensor theories.

$f(R)$ theories

- **Example:** R^2 gravity ($f(R) = R + aR^2$)

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-\tilde{g}}\left[F(\Phi)\tilde{R} - Z(\Phi)\tilde{g}^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - 2U(\Phi)\right] + S_m\left[\Psi_m;\tilde{g}_{\mu\nu}\right]$$

$$F(\Phi) = \Phi = f'(R)$$

$$2U(\Phi) = \frac{1}{8a}(\Phi - 1)^2 \;\Rightarrow\; m_\Phi = \frac{1}{\sqrt{6a}}$$

Field equations in STT (Einstein frame)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G_* T_{\mu\nu} + 2\partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}g^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi - 2V(\varphi)g_{\mu\nu}$$

$$\nabla^\mu\nabla_\mu\varphi = -4\pi G_* k(\varphi)T + \frac{dV(\varphi)}{d\varphi}.$$

These equations have to be supplemented with:

- Equation for hydrostatic equilibrium
- Equation of state of the nuclear matter

Equilibrium rotating neutron star solutions

Scalar-tensor theories with a massless scalar field

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \cancel{4V(\varphi)}\right) + S_m[\Psi_m; A^2(\varphi)g_{\mu\nu}]$$

Coupling function $\alpha(\varphi) = \frac{d\, lnA(\varphi)}{d\ \varphi}$

- The coupling function can be expanded as $\boldsymbol{\alpha(\varphi) = \alpha_0 + \beta\varphi +}$ **higher order terms**

  1. $\boldsymbol{\alpha(\varphi) = \alpha_0}$
    - Equivalent to the Brans-Dicke theory.
    - Differs from GR in the weak field regime.
    - Neutron stars have nontrivial scalar field for every $\alpha_0 \neq 0$

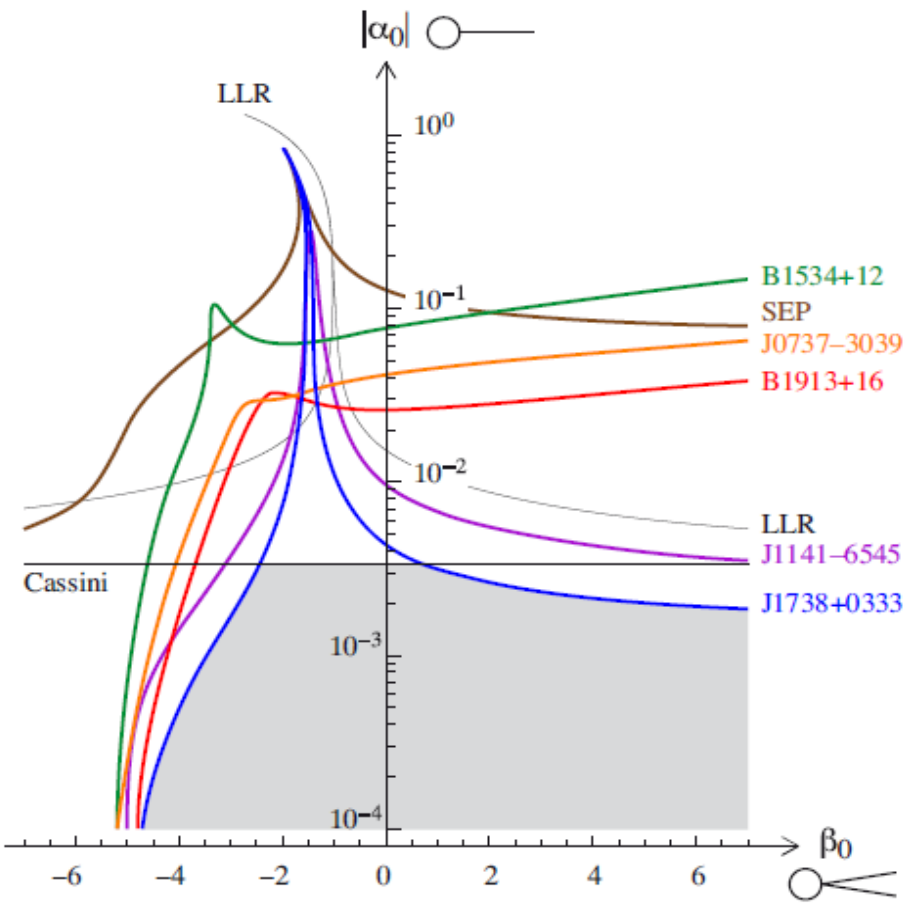
  2. $\boldsymbol{\alpha(\varphi) = \beta\varphi}$
    - Equivalent to GR in the weak field regime.
    - Can differ significantly when strong fields are considered.
    - Nonuniqueness of the neutron star solutions can exist – one solution with trivial scalar field and one or several others with nontrivial scalar field.

- **Higher order terms** in $\alpha(\varphi)$ lead to qualitatively similar results

Observational constraints

$\alpha_0 < 0.004$ and $\beta > -4.5$

(Damour & Esposito-Farese (1996,1998), Will (2006), Freire et al (2012), Antoniadis et al (2013))



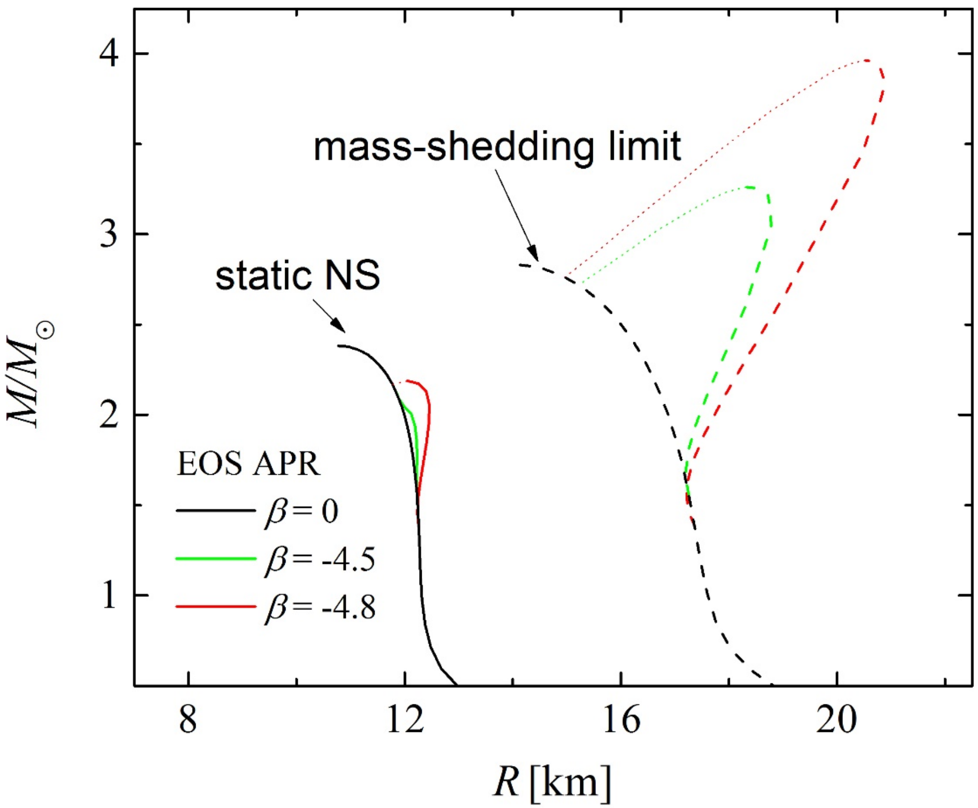
Freire et al (2012)

- Scalarized solutions exist only for $\beta < -4.35$ in the static case and $\beta < -3.9$ in the rapidly rotating case.

Equilibrium neutron star solutions: massless STT

- **Scalarization of neutron stars** in the second class of scalar-tensor theory was considered for the first time by Damour&Esposito-Farese (1993)
- **Slow rotation approximation** was also considered (Damour&Esposito-Farese (1996), Sotani (2012), Pani&Berti(2014)).
- **Rapid rotation** – changes the picture significantly (Doneva et al (2014))

Coupling function $\alpha(\varphi) = \beta\varphi$



- Scalarization possible also for **positive β** and negative trace of the energy momentum tensor. Possible for stiff EOS and very massive stars, not fully studied yet (Mendes (2015), Mendes&Ortiz(2016), Palenzuela&Liebling(2015))
- **Tensor-multi-scalar theories** (Horbatsch et al (2015)) – new interesting phenomena, still in development.

Scalar-tensor theories with a massive scalar field

- Neutron stars, with a massive scalar field could, in principle, have rather different structure and properties compared to their counterparts in the massless case.
- A new and promising line of research (Popchev Master Thesis (2015); F. Ramazanoğlu, F. Pretorius (2016), Yazadjiev, Doneva & Popchev (2016), Doneva &Yazadjiev (2016))

- The recent astrophysical and cosmological observations have severely constrained the basic parameters of the scalar-tensor theories with a massless scalar field leaving a narrow window for new physics beyond general relativity.

The situation changes drastically if we consider a massive scalar field.

- **The scalar field mass m_{φ} leads to a finite range of the scalar field of the order of its Compton wavelength $\lambda_{\varphi} = 2\pi/m_{\varphi}$.**
  - The presence of the scalar field will be suppressed outside the compact objects at distances $D > \lambda_{\varphi}$.
  - This means in turn that all observations of compact objects involving distances greater than λ_{φ} cannot put constraints, or at least stringent constraints, on the scalar tensor theories.

*P. Freire et al. (2012); Antoniadis et al. (2013); L. Perivolaropoulos, PRD **81**, 047501 (2010); J. Alsing, E. Berti, C. M. Will, and H. Zaglauer, PRD **85**, 064041 (2012); M. Hohmann, L. Järv, P. Kuusk, and E. Randla, PRD **88**, 084054 (2013); A. Scharer, R. Ang´elil, R. Bondarescu, P. Jetzer, and A. Lundgren, PRD **90**, 123005 (2014); L. Jarv, P. Kuusk, M. Saal, and O. Vilson, PRD **91**, 024041 (2015)*

$$S = \frac{1}{16\pi G_*}\int d^4x\sqrt{-g}\left(R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)\right) + S_m[\Psi_m; \mathcal{A}^2(\varphi)g_{\mu\nu}]$$

Coupling function $\alpha(\varphi) = \frac{d\ lnA(\varphi)}{d\ \varphi}$

- We shall consider **two** standard choices of the **coupling functions:**
  - **Brans-Dicke** coupling $\alpha(\varphi) = \alpha_0 \Leftrightarrow A(\varphi) = \exp(\alpha_0\varphi)$
  - Theory with **spontaneous scalarization** $\boldsymbol{\alpha}(\varphi) = \beta\varphi \Leftrightarrow A(\varphi) = \exp\left(\frac{\beta}{2}\varphi^2\right)$, where $\beta < 0$
- **The mass** of the scalar field – accomplished via **a nonzero potential** of the scalar field $V(\varphi) = \frac{1}{2}m_\varphi^2\varphi^2$

Massive Brans-Dicke theory

- For **massive Brans-Dicke** theory with $\boldsymbol{m_{\varphi} \geq 2\times10^{-14}eV}$ the Solar System observations cannot put constraints on the Brans-Dicke parameter α_0 and all values of α_0 are observationally allowed.

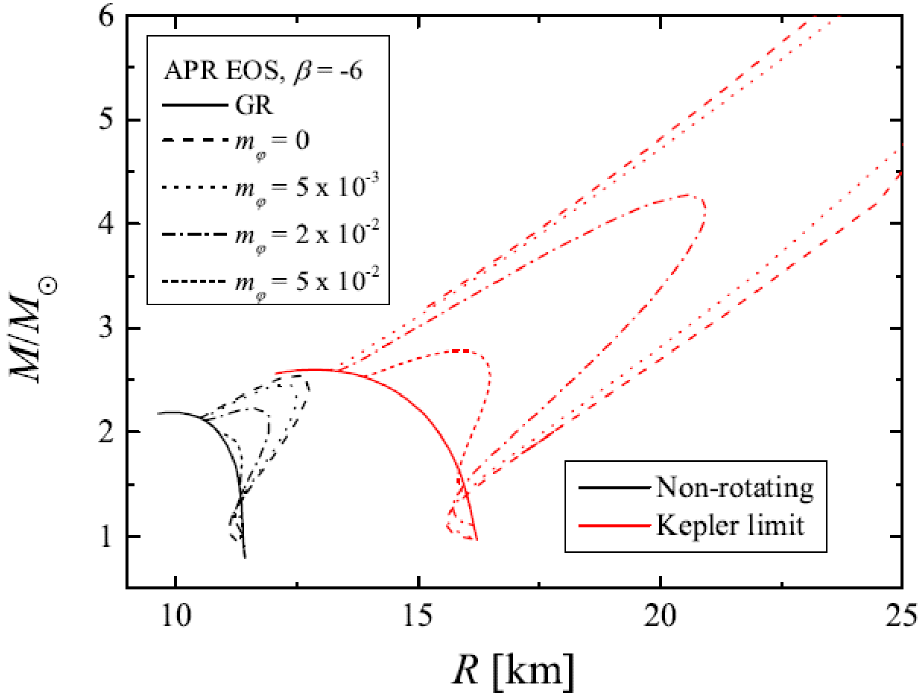
Scalar-tensor theory with $\alpha(\varphi) = \beta\varphi$

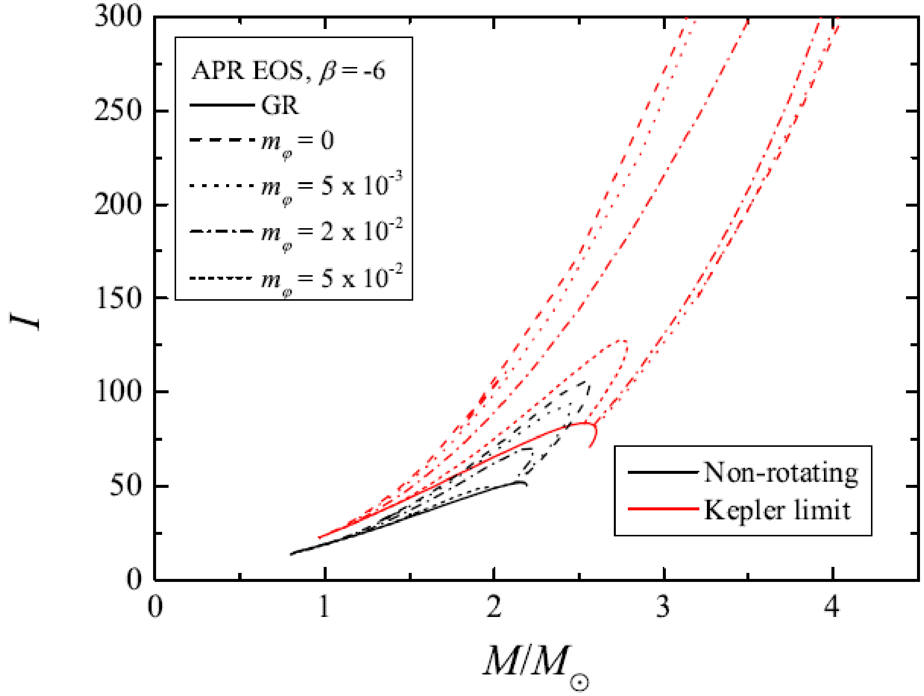
- The mass of the scalar field can effectively suppress the scalar gravitational waves and reconcile the scalar-tensor theories with the binary neutron star observations for a much larger range of β.
- If the Compton wave-length of the scalar field λ_{φ} is much smaller than the separation of the two stars in the binary system the emitted scalar gravitational radiation will be negligible $\boldsymbol{10^{-16}\mathrm{eV} < m_{\varphi} \geq 10^{-9}eV}$

F. Ramazanoğlu, F. Pretorius (2016); Yazadjiev, Doneva & Popchev (2016)

Theory with **spontaneous scalarization:**

$$\alpha(\varphi) = \beta\varphi \;\Leftrightarrow\; A(\varphi) = \exp\left(\frac{\beta}{2}\varphi^2\right), \beta < 0$$





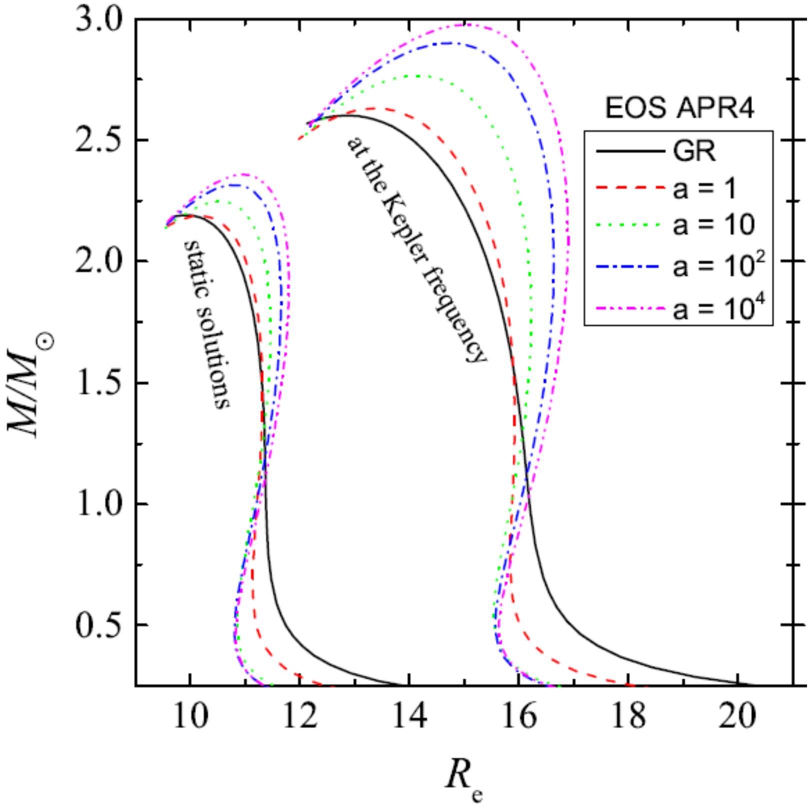
Yazadjiev, Doneva & Popchev (2016); Doneva &Yazadjiev (2016)

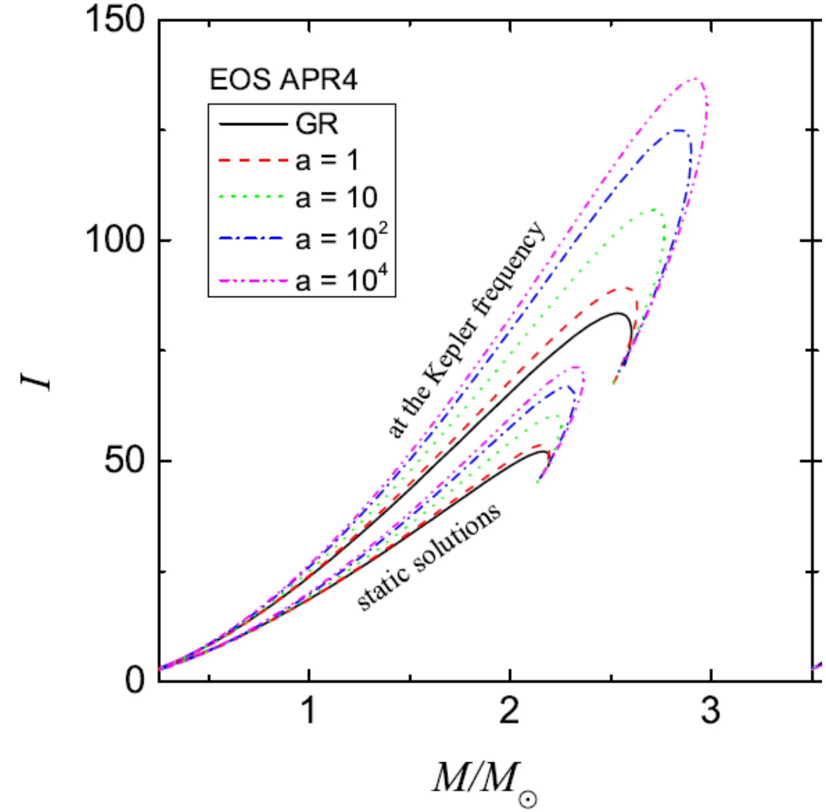
Equilibrium neutron star solutions: $f(R)$ theories

- **Non-perturbative approach**: reported in Babichev&Langlois(2010), Jaime et al (2011), and the first detailed study of realistic NS models was done in Yazadjiev, Doneva, Kokkotas, Staykov (2014)

- **Rotating models** are also studied (Staykov et al (2014), Yazadjiev et al (2015))

- **Non-negligible deviation** for the allowed values of a. The **moment of inertia** is very sensitive and can be used to set constraints on the parameters.





Oscillations and gravitational wave emission

Astrophysical implications

- **Final goal** – test the strong field regime of gravity via neutron star observations and impose constraints on the alternative theories

- **Obstacles**:
  - ➢ Accuracy of observations
  - ➢ Accurate models of the observed phenomena
  - ➢ EOS uncertainty

- Ways out:
  - ➢ Deviation from GR stronger than the EOS uncertainty for the allowed range of parameters
  - ➢ EOS independent relations

Possible approaches for testing alternative theories of gravity

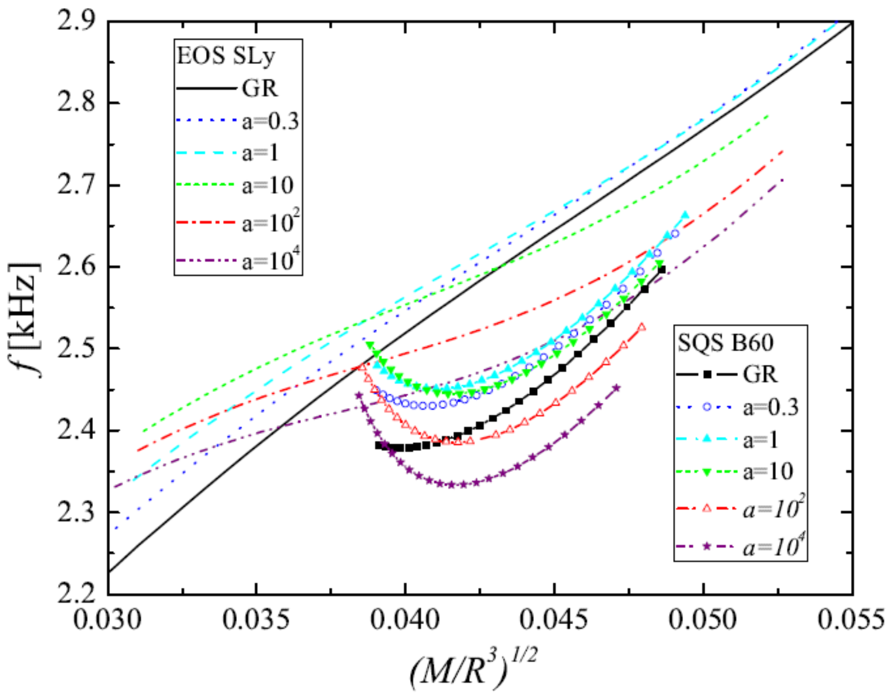
- Direct observation of the mass and radius.
- Observations of the moment of inertia: applicable for example for $f(R)$ theories Staykov at al (2014) and Eddington inspired gravity Pani, Cardoso, Delsate (2011)
- Quasiperiodic oscillations DeDeo&Psaltis(2004), Doneva et al (2014), Staykov, Doneva, Yazadjiev (2015)
- The redshift of surface spectral lines in X-rays and γ-rays DeDeo&Psaltis(2003)
- Gravitational wave emission of oscillating neutron stars
- Universal relations
- Neutron star mergers

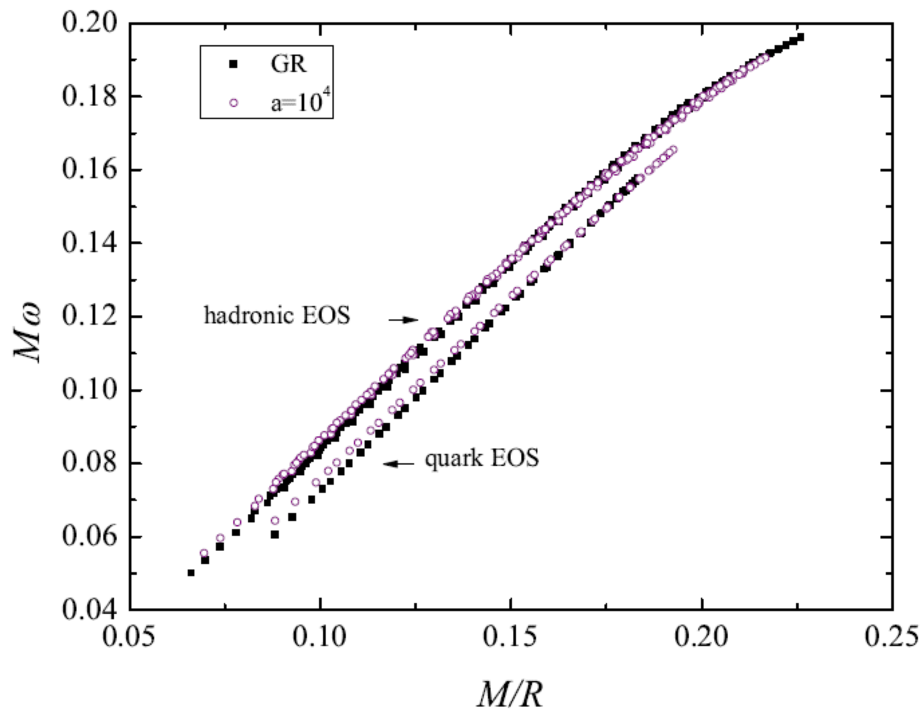
Neutron star oscillations

- The study was **initiated** with the work of Sotani&Kokkotas (2004) for *f*- and *p*-modes in STT.
- The main idea is to constrain the deviations from GR using the emitted gravitational wave signal or in some cases electromagnetic signal, related to neutron star oscillations
- Several alternative theories studied until now – STT Sotani&Kokkotas (2004), Silva et al (2014), TeVeS Sotani (2010, 2011, 2009), $f(R)$ Staykov et al (2015), Einstein-Gauss-Bonnet-dilaton gravity Blázquez-Salcedo et al (2016)
- Fundamental *f*-modes, torsional modes, w-modes and others are studied. In many cases the Cowling approximation is employed.

Asteroseismology relations in R^2 theories

- ***f*-mode** oscillation frequencies, nonrotating case
- Quite **EOS independent** with suitable choice of normalization

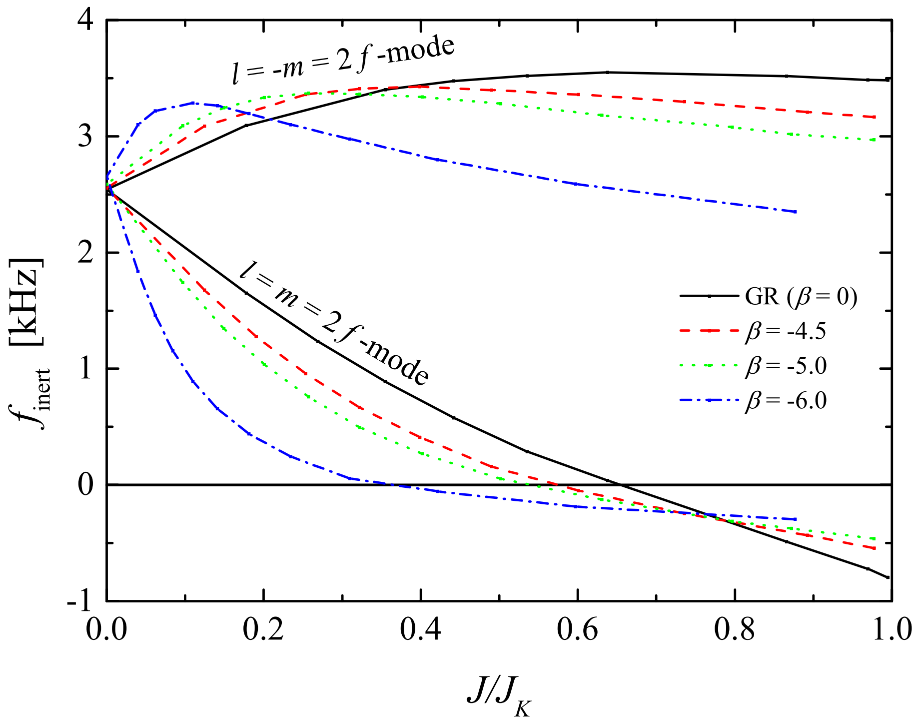


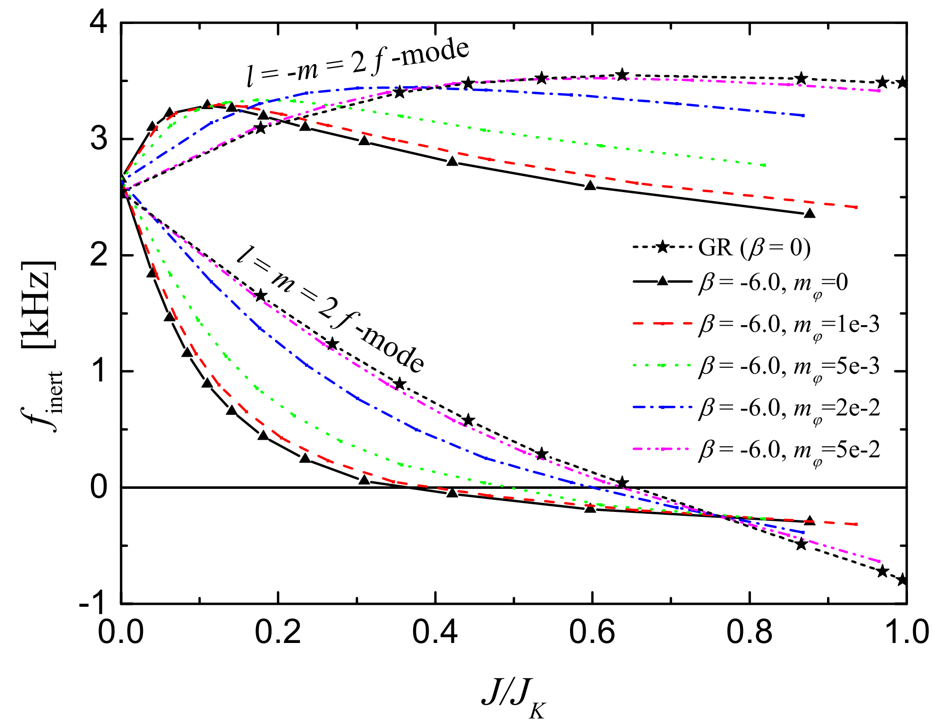


Staykov et al (2015)

Oscillations modes of rapidly rotating neutron stars

- ***f*-mode** oscillation frequencies, $l = |m| = 2$ case (prone to the CFS instability)





Yazadjiev, Doneva, Kokkotas (2017)

Conclusions

- Neutron stars in alternative theories of gravity can have significantly different properties compared to their general relativistic counterparts and the rotating magnifies these differences significantly.

- The oscillation modes and the related gravitational wave emission can be used as a test of the alternative theories of gravity.

- Further info: Berti et al. (2015)

Thank you!