The Features of the Spacetime

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Introduction

In the last ten years two very significant experiments have occurred: The LIGO Laser Interferometer Gravitational-Wave Observatory (there are twin LIGO detectors at Livingston Louisiana and Hanford Washington) and the Gravity Probe B (GP-B), both of them dealing with the General Relativity (GR) theory. LIGO is an experiment concerning with the existence of the gravitational waves (GW), a phenomenon which was anticipated by Einstein decades ago. GPB has aimed to put into evidence how the Earth's rotation drags the local spacetime's frame with it (frame dragging was foresighted by Einstein as well).

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These two experiments were thought many years ago, the only one reason they were not achieved was the technological unsatisfactory level. It was needed a high accuracy because the expected experimental data was very small. For LIGO, it was expected to be finding a change in the arm length (the arm length of the GW detector is 4 km) of the order of 10^{-17} cm. For GP-B they have struggled to measure the drift of the orientation of a gyroscope’s spin axis of 6,614.4 milli-arc-seconds per year in the orbital plane of the satellite caring the experiment, due to the curvature of Earth’s local spacetime, and the drift of the spin axes of 40.9 milli-arc-seconds per year in a perpendicular plane (that is, the plane of Earth’s rotation), due to the frame-dragging effect. In order to visualize such a little angle you can imagine how little we see, from one side to the other, a strand of hair, from thirty kilometers away. Another important commune features for LIGO and GP-B is how expensive they are and the fact that they are at cosmic scale. The key players were Kip S. Thorne (Caltech) for LIGO and Leonard Schiff (Stanford) for GP-B and both experiments have led to a Nobel Prize.

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LIGO and GP-B undoubtedly prove that the space time is not anymore only a mathematical tool but it is rather, a real physical object featuring peculiar properties. The space time is “elastic” in the sense of being a proper environment considering the existence of the gravitational wave. The spacetime is “viscous” in the sense that it could be dragged by another material body moving inside.

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- The notion of gravitational force is replaced by the curvature of the spacetime fabric.
- “The mass tells to spacetime how to curve and the curved spacetime tells to the body bearing this mass, how to move”.
- We have to take into consideration the elastic and the viscous nature of the spacetime fabric.
- The most revolutionary contribution Einstein had, to science, was formulating this statement:

  the spacetime’s fabric is elastic…and viscous.

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The space time is granular

The viscoelastic feature of the spacetime is emergent. It appears because the spacetime has to be granular and the granularity at big numbers imply a statistical behaviour leading to the viscoelasticity. I think that the humanity doesn't know the complete landscape of the divergent series which is the key of the emergent phenomena. Nevertheless, the General Relativity Theory has introduced a dipper understanding of the fundamental notion of the spacetime. The spacetime is a real physical object, a manifestation of some real field. As any physical field, the spacetime has to manifest quantum features at proper sc ales.

During the historic meeting between Lev Landau, Niels Bohr, Leon Rosenfeld and Matvei Bronstein in Moscow, 1934, when they have introduced the idea of the quantum gravity, some others significant ideas were discussed. Indeed, considering the scattering of the neutral particles, the General Relativity Theory is good enough to describe the phenomenon at high energy (considering the energy in the CM reference system), for some normal values for the impact parameter. Otherwise, for low energies in the CM reference system and normal values for the impact parameter, The Quantum Field Theory is effective in the prediction of the scattering amplitude. Quantum Gravity has to deal with the remaining large domain between the two very successful theories of physics . The question is if in the new, eagerly awaited theory we can have an arbitrary precise localization in spacetime since into a certain point, the Heisenberg uncertainty principle, does occur. Indeed, Bronstein has proven that in the frame of the general relativity and because of the quantum principles is impossible to measure the gravitational field into an arbitrary small region. This assumption is very important for a proper quantum gravity theory, we are expecting to come. A shallow proof of the assumption, above is presented in the next paragraphs.

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Let's consider a location x where we intend to measure a field value. If you want to determine the location with the precision D, you may do it using a particle at x. Since the particle is quantum, we have Δx and Δp, the uncertainties corresponding to the position and the momentum of theparticle. So, Δx has to be smaller than D and greater than $h/2\pi\,\Delta p$ or **$\Delta p > h/2\pi D$**. The mean value of p^2 is greater than $(h/2\pi D)^2$ which is an important consequence of Heisenberg uncertainty:"**you need large momentum in order to put into evidence a sharp location**". At relativistic speeds, you are allowed to consider the rest mass as negligible and take the energy $E \sim cp$. In the text above, it was about quantum physics and now we take into consideration the general relativity, where any form of energy is actually a gravitational mass $m \sim E/c^2$. A gravitational mass bands the spacetime in a rate proportional to its value. If the mass is contained in a sphere with the radius **$r \sim Gm/c^2$**, then a black hole emerges. Aiming to get a sharper localization, we consider a very small D. So, the energy increase till r is biger than D. the unavoidable conclusion is that the domain D, we want to mark, will be hidden beyond the black hole horizon and the localization is lost. There is a minimum value for D, meaning D=r. The final conclusion is that a particle which is quantum may be localized for $D = mG/c^2 = hG/2\pi Lc^3$. The precision of the localization for a certain object does not exceed the Planck length **$D = \sqrt{hG/2\pi Lc^3} = 10^{-33}$cm**.

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The spacetime is smooth only above this length. Bellow it, the notion of distance is senseless. We need a quantum geometry. We talk about quantum states of the spacetime not quantum states on spacetime.

All these are good arguments to consider the spacetime as granular. Considering the granular nature of the spacetime we have to take into account the quantum scientific method. If the space time has quantum nature then it has to be possible to do a **discretization** on it; The uncertainty principle imply a **fuzziness** of the spacetime atoms; the structure contains spacetime atoms, dynamical connected each other and their **evolution has a probalistic nature**. Graphs, where spacetime atoms are the nodes and the links are the field lines of the gravitational fields, represent the fundament of the **Loop Quantum Gravity**. This theory features the braid and the knot theories. The later is the main subject in the next section of this paper.

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The spacetime and knots

In extreme physical condition (a large magnetic field and a very low temperature) some special materials behave peculiar. The quantum Hall effect does appear in such special systems. New physical objects emerge: **anyons**. In the sense of the number of particles allowed to occupy certain quantum levels, they are nor fermions, neither bosons. Actually they are quasi particles, leaving in 2+1 spacetime. They don't obey the Bose-Einstein statistics (the bosons may simultaneously occupy the same quantum states) and they don't obey the Fermi-Dirac statistics (two fermions are not allowed to occupy the same quantum state).In the special case of the anyons, the statistics they obey is named the fractional statistic.

The peculiar values for the temperature and the magnetic field determine a two dimensional surface where the anyons do exist. In the theoretical approach of Robert B. Laughlin (1998 Nobel Prize for discovering that electrons acting together in strong magnetic fields can form new types of "particles", with charges that are fractions of electron charges) this surface is a plateau where the anyons move chaotic. If we resume only to a couple of anyons then they rotate one around the other. Now, if we consider two consecutive moments in time, they create a braid. Into the mathematical braid theory (the braid algebra), a braid is intimately connected to a certain mathematical knot. Consequently, the dynamics of the anyons is governed by the knots theory, the latter being the necessary tool for quantum computation (which in this case is named "the topological quantum computation" and a topological quantum computer is the host device of such computation). In the last thirty years, some state of the art experiments aiming to provide a two dimensional environment for the quasi particles dynamical evolution, were developed:

- MOSFET (Metal Oxide Semiconductor Field Effect Transistor) using the effect of the inversion layer
- Super lattice which is basically a two dimensional electron system that appears in the heterostructures of two semiconductors
- Liquid Helium Surface where two contrary actions coexist, constraining the particles to remain in two spatial dimensions (the potential barrier and the mirror potential).

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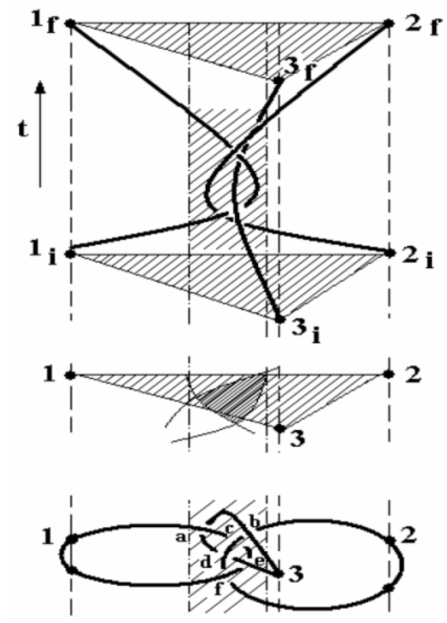
The quantum knots

If the anyons' trajectories are not so closed, then the corresponding braid and the resulting mathematical knot are well defined. In this case we have no doubt about the number of crossings and about the type of the knot, the anyons conceive. But, if the anyons' trajectories are closely tight then they surpass a threshold and the type of the corresponding knot become not so well defined. The knot behaves like a quantum object and it can be of one type or another, with some probabilities. A certain knot may occupy different states, each of them characterized by a certain probability, in the same manner a quantum particle does.



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The **Laughlin's function** was introduced (on the extreme quantum limit in which the Landau's level hierarchy is large enough and allows the agglomeration of all the anyons on the lowest level) as a single particle wave function:

$$\psi_m(x) = C \cdot |x|^m \cdot e^{-\frac{1}{4}|x|^2}$$

where "m" is an odd integer.

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In order to avoid the unpleasant divergences, I cut off the function and I have considered a finite radius R.

When the shaded areas (projections of Laughlin's functions) are clashing, the quantum feature emerges.

In Fig.2a, we can see the clashing of the functions, describing the corresponding distribution of probability, represented in three dimensional space.

In Fig. 2b, the same interaction is represented as seen by a bird eye with level curves.

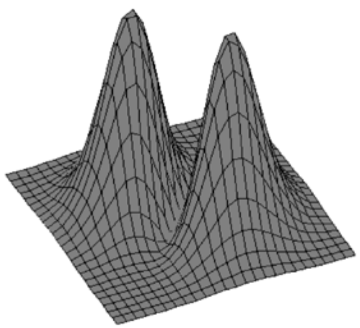
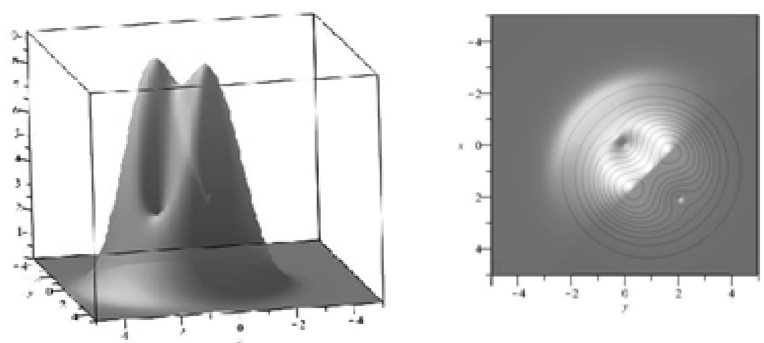


Fig. 2a. Two anyons, just before colliding each other



- **Fig. 2b**. Two anyons after colliding (two different views)

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In Fig. 3 we have represented the projections during the interaction. L is the distance between the anyons and it becomes a parameter into the function of probability gives us the probability to have the positions of anyons (left to the right) 1-2 or 2-1, which is dramatically different because it determines, in this way, what kind of crossing of trajectories we have.

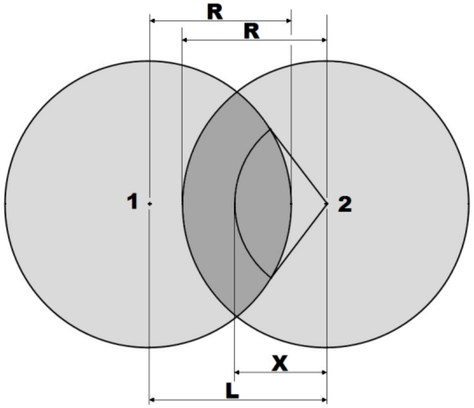

Fig. 3. Two anyons after colliding each other with L being the distance between them

I have calculated the probability function for a crossing to be “knotted” or “unknotted” departing from the Laughlin function. So, “the crossing integral” P (actually the probability to interchange the anyons’ trajectories) is:

$$\mathrm{P(L)} = \frac{1}{\pi}\int_{\mathrm{L}-1}^{1} |\psi_3(x)|^2 \cos^{-1}\frac{|x|^2 + L^2 - 1}{2L|x|}\, dz$$

where R = 1and m = 3. For instance, if we take 0.5R for L then the probability to have a crossing is P=0.2567.

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Three crossings represent a "quantum" knot Qk_3 which is a three foil knot with probability P or an unknot with the probability 1-P:

$$Qk_3 = Pk_3 + (1-P)k_0$$

Hence, when we consider the evolution of some anyons we have to take into consideration the "L" parameter of vicinity during the crossing and consequently we will have simultaneously, not only a single and well defined knot but a "quantum" knot represented by many possible knots, everyone with its specific probability of existence. The real one, will emerge only after the measurements on the type of crossings will, be accomplished. Nevertheless the order of the knot has to be less or equal to the number of crossings. Therefore, we have to consider the possibility of laddering an entire part of the pattern of the braid.

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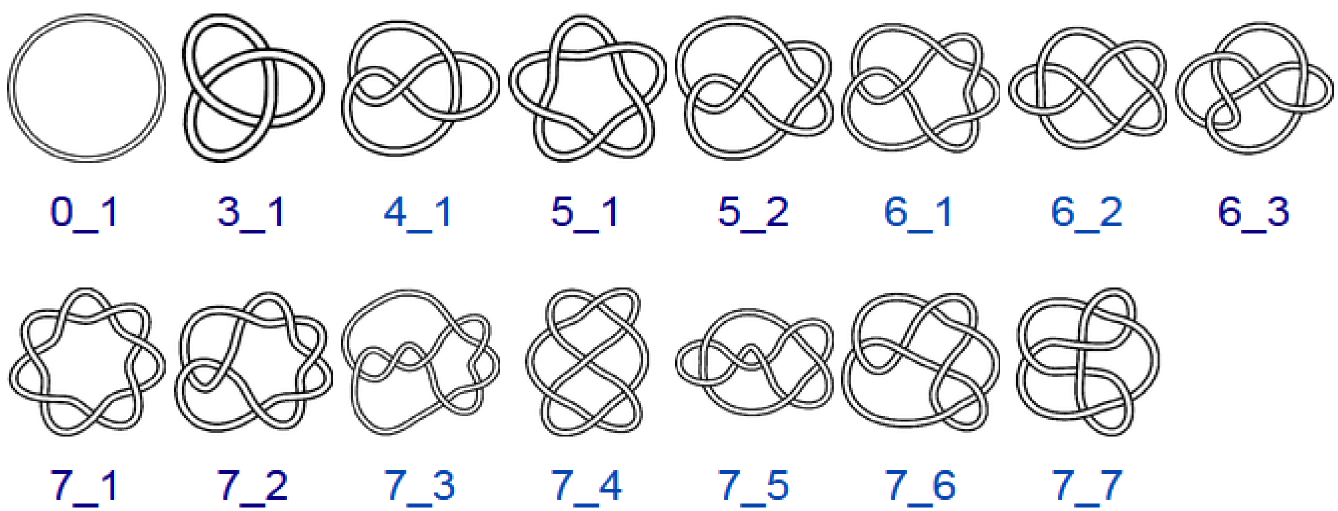
n	1	2	3	4 5 6	7	8	9	10	11	12	13	14	15	16
Number of prime knots with *n* crossings	0	0	1	1 2 3	7	21	49	165	552	2176	9988	46972	253293	1388705
Composite knots	0	0	0	0 0 2	1	4	...		...		...		...	
Total	0	0	1	1 2 5	8	25	...		...		...		...	

Enantiomorphs are counted only once in this table and the following chart (i.e. a knot and its mirror image are considered equivalent).



Knots with 7 or fewer crossings

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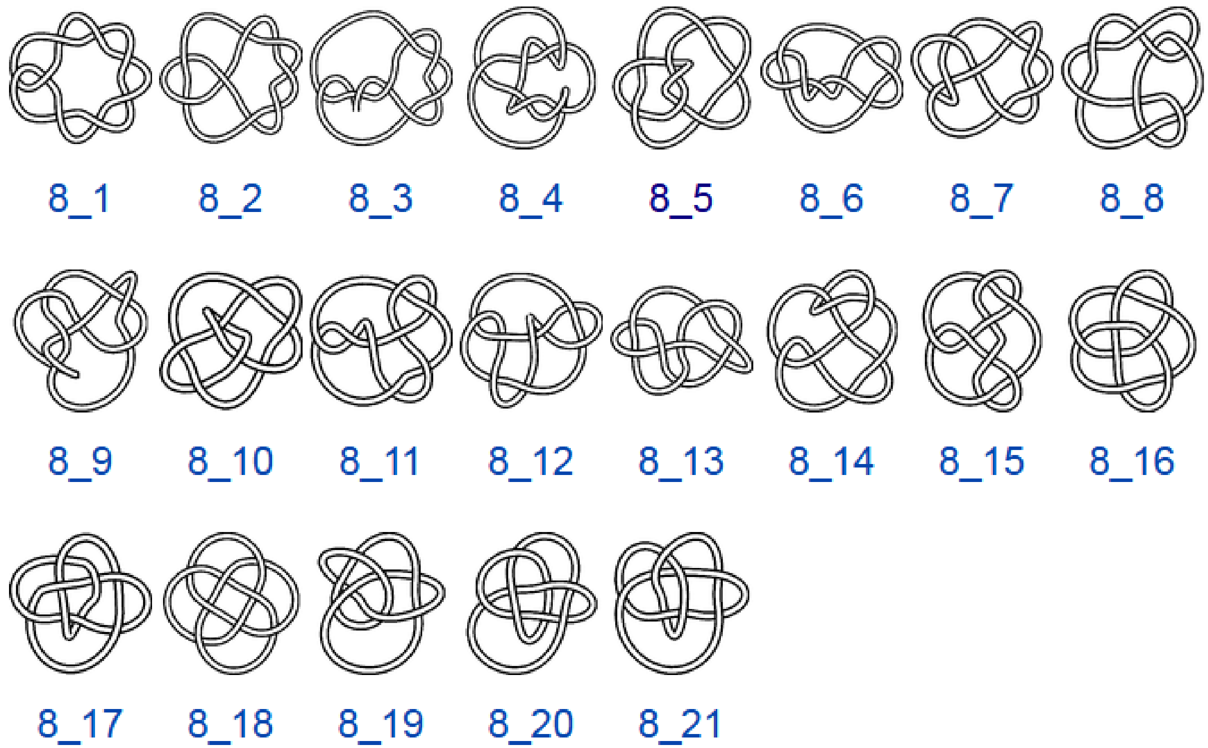

Knots with 8 crossings

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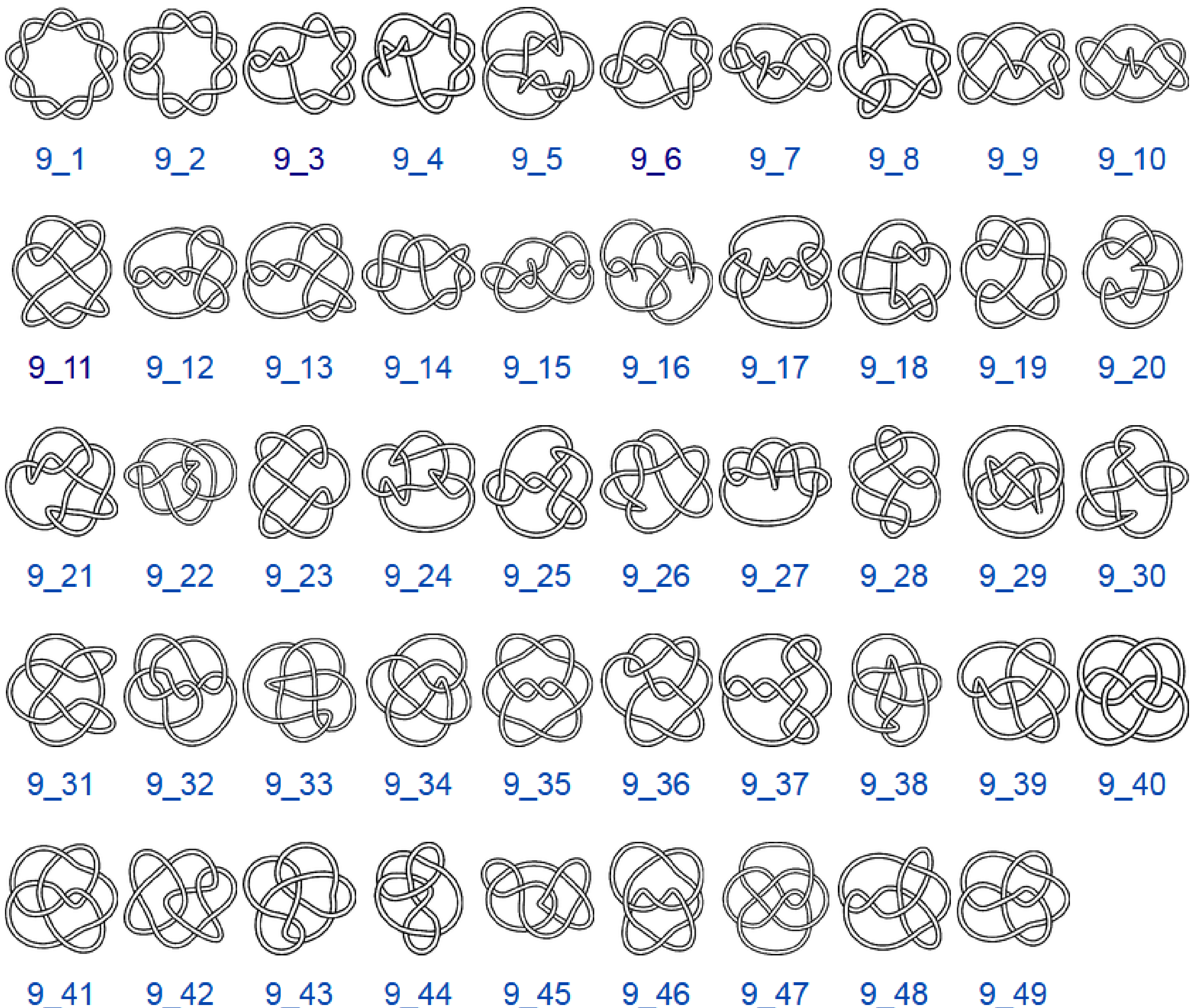

Knots with 9 crossings

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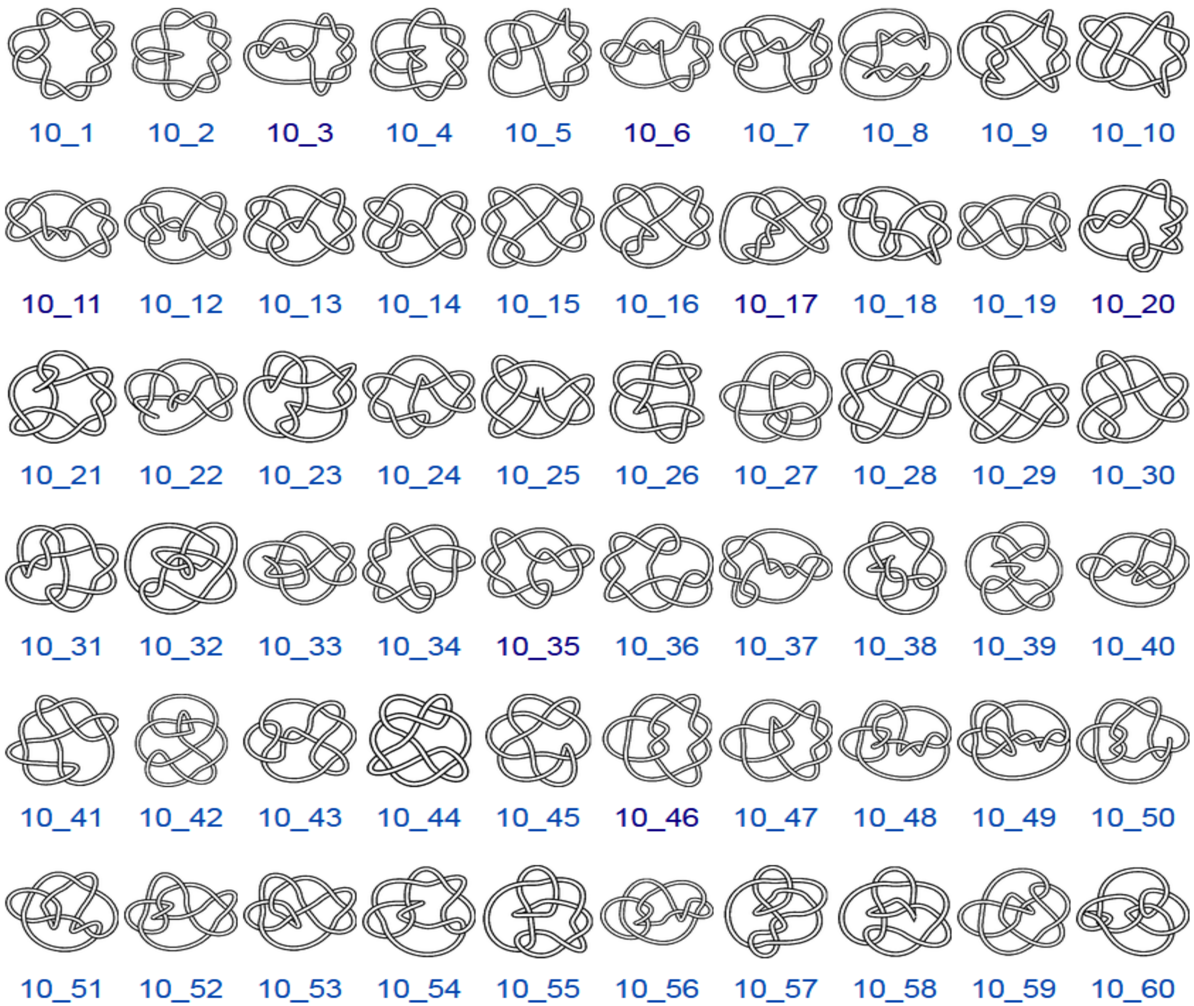

Knots with 10 crossings (1-60)

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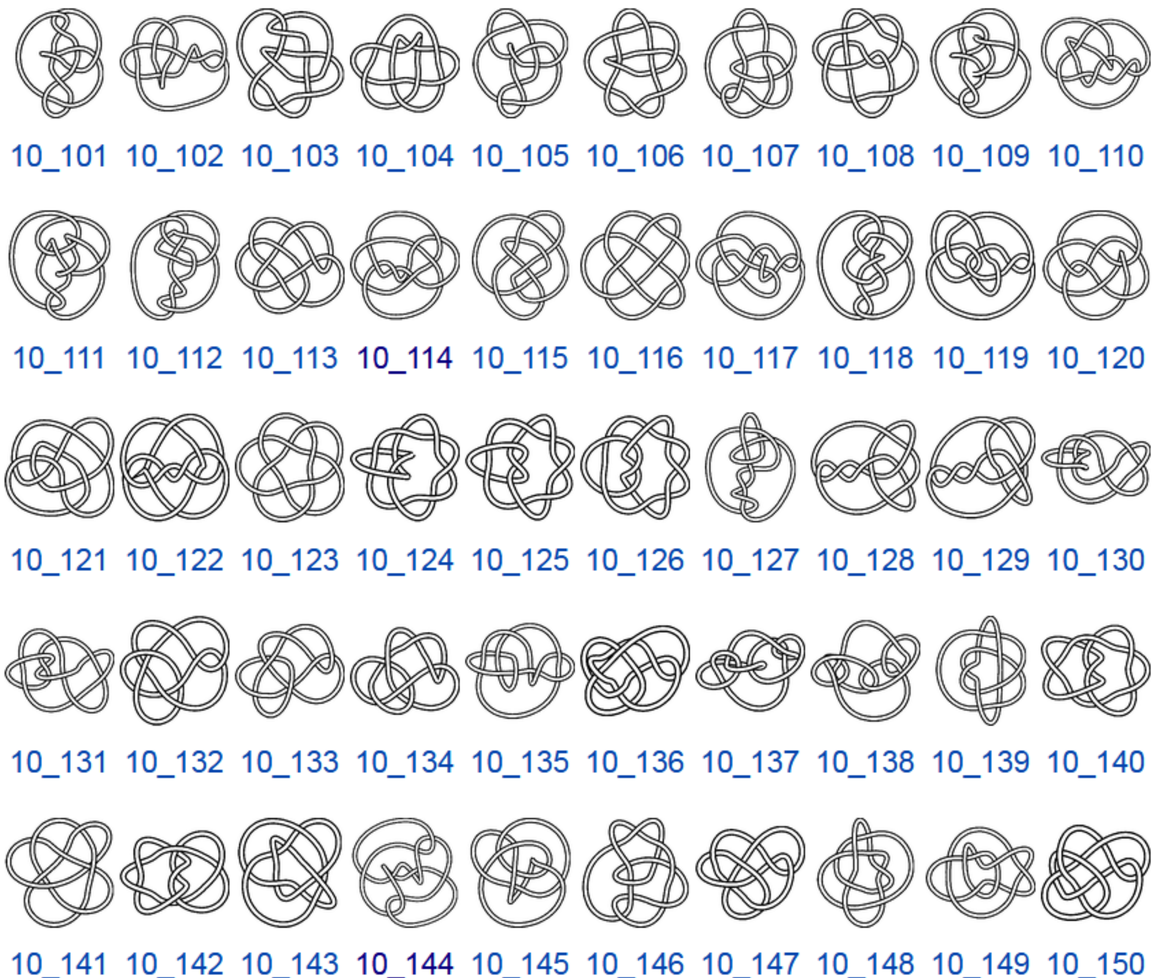

Knots with 10 crossings (101-150)

The "quantum" knots and the ladder operators acting on the knot's subfield

Let's pick randomly a knot from **the Rolfsen atlas** of prime knots [5],[6]. This atlas is going only to the ten crossing knots. In Fig. 4 I have chosen the 9.6 knot. In Fig. 4a is depicted the Rolfsen representation. In Fig. 4b I have represented the same knot using a simple but a proper diagram, which is different, considering even the Morse diagram. Every elementary box represents a direct crossing 

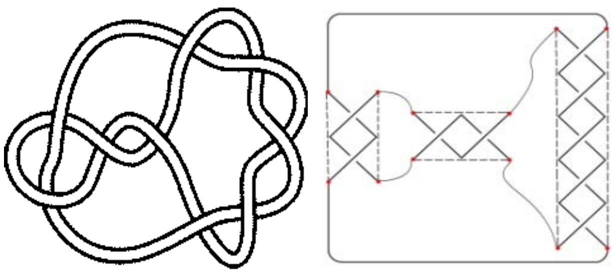

Fig. 4a. The 9.6 knot

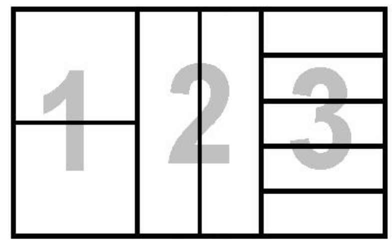

Fig. 4b. The 9.6 diagram

We can separate three different areas on this 9.6 knot diagram, considering their topology. Let's call them, subspaces and label them by numbers. The subspaces could be vertical or horizontal. The structure of a given knot is complex.

Its main feature is the summation of corresponding brackets and every bracket may contain an infinite number of brackets dispose in every way they are allowed. Therefore we have an infinite number of terms and these terms are contained into a bracket, containing another bracket, containing another bracket and so on. We are numbering all these subspaces going to the infinity. If the knot is finite, then we may cut off, considering all the brackets from "n" to infinity as "vacuum states". For instance, for 9.6 the labeling stops to three but we still have an infinite number of "zero" or "vacuum" subspaces. In the present case we have three, not zero base vectors: $|1,0,0>, |0,1,0>, |0,0,1>$, corresponding to the first, the second and the third subspaces. The knot state is:

$$2|1,0,0> +2|0,1,0> +5|0,0,1> = |2,2,5>$$

We introduce now, **the ladder operators** $\mathbb{O}^{+}$ $\mathbb{O}^{-}$

the creation one, which, every time it is applied it adds a new direct crossing 
the annihilation one, which, every time it is applied it adds an indirect crossing 

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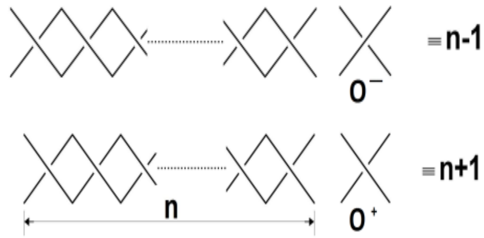
We define the Bose ladder operators according to:

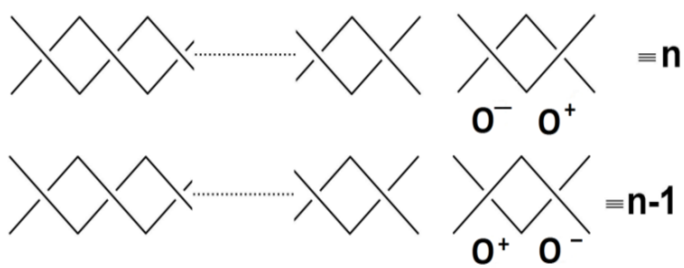
$$\mathbb{O}_i^{\,-}|n>=\sqrt{n}\;|n-1>$$

$$\mathbb{O}_i^{\,+}|n>=\sqrt{n+1}\;|n+1>$$



These creation and annihilation operators obey the Heisenberg algebra:

$$[\mathbb{O}_i^-,\mathbb{O}_j^-]=[\mathbb{O}_i^+,\mathbb{O}_j^+]=0 \qquad [\mathbb{O}_i^-,\mathbb{O}_j^+]=\delta_{ij}\mathbb{I}$$



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For the 9.6 knot, next equations are valid:

$$(\mathbb{O}^-)_1{}^2 k_{9.6}^1 = k_5$$

$$(\mathbb{O}^-)_2{}^2 k_{9.6}^2 = k_{7.2}$$

$$(\mathbb{O}^-)_3{}^5 k_{9.6}^3 = l_2$$

the annihilation operator may acts on a certain **Conway knot polynomial** providing the corresponding one

$$(\mathbb{O}^-)_1{}^2(2z^6 + 8z^4 + 7z^2 + 1) = 2z^2 + 1$$

$$(\mathbb{O}^-)_2{}^2(2z^6 + 8z^4 + 7z^2 + 1) = 3z^2 + 1$$

$$(\mathbb{O}^-)_3{}^5(2z^6 + 8z^4 + 7z^2 + 1) = z^2 - 1$$

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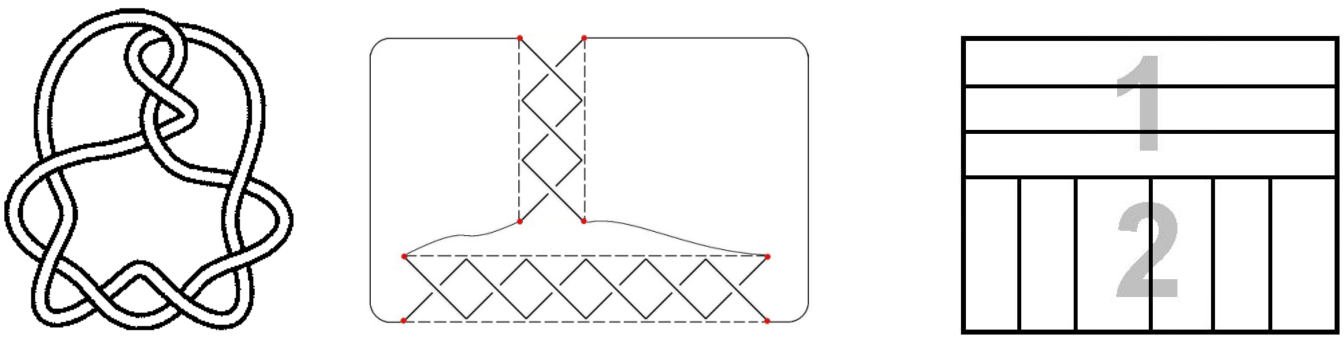

Fig. 5. The 9.3 knot with the coresponding diagram

$$(\mathbb{O}^-)_1{}^3 k_{9.3}^1 = k_0$$

$$(\mathbb{O}^-)_2{}^6 k_{9.3}^2 = k_0$$

$$(\mathbb{O}^-)_1{}^3 (2z^6 + 9z^4 + 9z^2 + 1) = 1$$

$$(\mathbb{O}^-)_2{}^6 (2z^6 + 9z^4 + 9z^2 + 1) = 1$$

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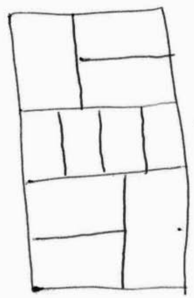
$K_{10.144}$



$K_{10.144}$ diagram

$\mathbb{O}_{1}^{-1} K_{10.144}^{1} = K_{7.1}$

$\mathbb{O}_{2}^{-2} K_{10.144}^{2} = K_{7.1}$; $\mathbb{O}_{2}^{-1} K_{10.144}^{2} = K_{9.11}$

$\mathbb{O}_{3}^{-4} K_{10.144}^{3} = K_{6.3}$

Conway polynomials:

$\mathbb{O}_{1}^{-1}(-3z^4-2z^2+1) = z^6+5z^4+6z^2+1$

$\mathbb{O}_{2}^{-2}(-3z^4-2z^2+1) = z^6+5z^4+6z^2+1$

$\mathbb{O}_{2}^{-1}(-3z^4-2z^2+1) = -z^6-z^4+4z^2+1$

$\mathbb{O}_{3}^{-4}(-3z^4-2z^2+1) = z^4+z^2+1$

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When we drain a subspace till it has no boxes in it we have, what we call, a vacuum state $|0>$

$<0|0>=1$ -the probability to have a vacuum state into an empty box, is one.

a vacuum state is defined by $\mathbb{O}^{-}|0>=0$ (an empty box into the diagram is erased)

You can, permanently create a "many-particle" state by acting repeatedly with creation operators on the vacuum state (create many crossings into the empty subspaces of the knot). We have to introduce a new rule: the creation operators act always to the left of the annihilation operators (normal ordering using the symbol : :):

$$:\mathbb{O}_1^{+}\mathbb{O}_2^{-}\mathbb{O}_3^{+}\mathbb{O}_4^{-}: = \mathbb{O}_1^{+}\mathbb{O}_3^{+}\mathbb{O}_2^{-}\mathbb{O}_4^{-}$$

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Concluding remarks

The granular spactime determines the emergent viscoelasticity. The granular spacetime dynamics is knotted. The knots are quantic. The necessary mathematics is a special algebra of the ladder operators.

The idea of defining ladder operators on the frame of the knots is conspicuous. It generates a great liaison to the Bose theory. Being a fruitful idea it will find its way especially on the low topology theory. The study of the action of the ladder operators in the frame of the knots is a work in progress and I hope that in the very near future, the dynamics of the quantum knots will be a modern topic.

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Thank you for your time!

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