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The many faces of elastica. (English) Zbl 06755677

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This book is addressed to students, Ph.D. students and postdocs interested in the applications of differential geometry. The main investigations concern the equation $\dot{k}^2 = P_4(k)$, k is the curvature of the plane curve, $P_4(k)$ is a fourth-degree polynomial with real coefficients. Such polynomial can be found in the studies of the shape of elastic cylindrical membranes under pressure, and the equation is known as equation of the generalized elastics. The book consists of 6 chapters. In Chapter 1 main definitions and results from differential geometry of curves and surfaces are presented in connection with membrane shapes. Some results from variational calculus, as Euler-Lagrange equation are presented as well. Chapter 2 deals with Serret curves. Frenet-Serret equations are studied as dynamical systems. The methods of investigation are related to Euler elastic. This new technique allows the Sturm spirals and their generalizations, the Serret curves and their generalizations to be parametrized explicitly. Next chapters 3,4,5,6 are devoted to the study of biological membranes. Chapter 3 is introduction to the membranology. Chapter 4 deals with the equilibria of the membranes from mechanical point of view. The equilibrium surface is parametrized explicitly via elliptic integrals of first and second kind. In Chapter 5 the Canham and Ou-Yang and Helfrich models of membranes are studied as an extensions of Euler's elasticas from analytical point of view. In Chapter 6 explicit solution for several models are conctructed. Mathematical models of Cole experiment is presented as well.

Reviewer: Angela Slavova (Sofia)

MSC:

74B99 Elastic materials
53A04 Curves in Euclidean space
53A15 Affine differential geometry

Keywords:

differential geometry; elastica; surface; euler elastica; membranes

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