MR3699576 53A04 53-01 53A05 82D25 92C20 92C37
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★**The many faces of elastica.**
Forum for Interdisciplinary Mathematics, 3.
Springer, Cham, 2017. *xv+212 pp. ISBN* 978-3-319-61242-3; 978-3-319-61244-7

This book provides a treatment of a beautiful area of mathematics and its applications which ties together aspects of the classical differential geometry of surfaces and the calculus of variations. The many problems/applications presented in the book are representative of the underlying literature which includes the study of ordinary differential equations involving the spatially prescribed curvature of a planar curve. Physical phenomena from the shapes of hanging chains, bending beams, rotationally symmetric interfaces of liquids and bubbles are discussed, as well as more complicated versions of these in which various inhomogeneities are introduced.

Given the foundation of the calculus of variations and the current advanced stage of development, the mathematical modeling and treatment of these problems provide, certainly, one of the most well-motivated and compelling connections between mathematics and physical phenomena. The reader will find in this book a useful introduction to some of the relevant underlying mathematics; there is a nice introduction to the differential geometry of curves and surfaces and certain aspects of the calculus of variations. From there, the authors provide an introduction to various special curves and generalizations of many of them. A connection is then made between these tools and simplified versions of the biology and chemistry of membranes. What then follows is a somewhat free-flowing mixture of model descriptions for various problems/applications and mathematical analysis of those models (to varying degrees of development).

Some general observations about the nature of these problems and models may be useful to the reader. Most of the physical problems, especially the more exotic ones involving biological membranes, dynamic interfaces, and complicated deformation, probably involve important aspects which are not captured (or even contemplated) by the mathematical models presented. On the other hand, the models are inherently nonlinear and global in nature; they involve a variety of special functions, elliptic integrals, and related constructions. This imposes a certain level of complexity and difficulty in regard to the mathematical analysis of the models, especially if one seeks a very deep understanding of the mysteries held in the mathematics. As a result, what you have presented here exists in a kind of region of balance in several respects. If one seeks to include more exotic behavior, the ability to say very much of interest about the resulting mathematical model evaporates. If one simplifies the model, the (sometimes already tenuous) connection to the physical system is lost. Even some of the mathematical analysis related to the modeling can be, on the one hand, an artifact of the oversimplified nature of the model or, on the other hand, simply motivated by the beautiful (but artificially introduced) mathematical structure. It is partially for these reasons that the beautiful work presented here is often considered "very applied" by some mathematicians and "very pure" by others, especially because asymptotics and perturbations are largely neglected. The authors have provided a nice introduction to walking (or dancing) upon this particular fence and a seeming invitation to reject the greener grass on either side. *John McCuan*