SAXON–HUTNER THEOREM – A theorem concerning energy gaps in one-dimensional random alloy models described by the **Schrödinger equation** or the **Dirac equation**. It states that forbidden energies that are common to a pure crystal A and a pure crystal B (with the same lattice constant) will always be forbidden energies in any arrangement of the atoms of A and B in a substitutional solid solution.

Making use of the one-to-one correspondence between the real localized atomic potentials and (2×2)-transfer matrices belonging to any of the isomorphic three-dimensional Lie groups $\mathrm{SU}(1,1)$, $\mathrm{SL}(2,\mathbf{R})$ or $\mathrm{Sp}(2,\mathbf{R})$, this can be described in mathematical terms as follows. If $A^{r_1}B^{s_1}\cdots A^{r_k}B^{s_k}$ is an arbitrary linear chain consisting of two types of atoms A and B, each having representatives $r_i, s_i \in \mathbf{Z}^+$ in the ith period. Then the group nature of the individual transfer matrices M_A and M_B makes it possible to express the total transfer matrix M of the elementary cell as the product $M_B^{s_k}M_A^{r_k}\cdots M_B^{s_1}M_A^{r_1}$, and the forbidden energies for electrons propagating there are given by the condition $|\mathrm{tr}(M)| > 2$.

In the transfer-matrix approach, the theorem takes the form of the following question: Given that

$$|\mathrm{tr}(M_A)|, |\mathrm{tr}(M_B)| > 2,$$

is it true that for any arrangement $A^{r_1}B^{s_1}\cdots A^{r_k}B^{s_k}$ of atoms of type A and B one has

$$\left|\mathrm{tr}(M_B^{s_k}M_A^{r_k}\cdots M_B^{s_1}M_A^{r_1})\right| > 2\,?$$

Relying on quite different techniques, several non-equivalent sufficient conditions guaranteeing its validity have been derived.

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