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BOOK REVIEW

Geometric Phases in Classical and Quantum Mechanics, by Dariusz Chruściński and Andrzej Jamiolkowski, Birkhäuser, Boston·Basel·Berlin, 2004, xii + 344 pp. 47 illustrations, ISBN: 0-8176-4282-X

This new textbook examines one of the most interesting findings of the last several decades – the physical importance of the concepts of the topological and geometrical phases in classical and quantum mechanics.

The activity in this field has flourished ever since Berry published his seminal paper in 1984. In this paper Berry demonstrated that the conventional description of the adiabatic quantum evolution was incomplete as it ignored the additional phase factor (since then known as the Berry phase factor). Soon afterwards B. Simon observed that the Berry phase factor can be interpreted as a purely geometric object, namely as a holonomy in a certain fibre bundle (the spectral bundle), which is determined by the spectral properties of the Hamiltonian. It was realized also that the concept of the Berry-Simon geometrical phase finds fruitful applications in various fields of contemporary physics and chemistry – from Foucault pendulum, molecular physics and optical fibers to instantons and the quantum Hall effect.

The number of research publications on this topic has grown at such a pace that, for non-experts, it has become difficult to follow the numerous theoretical ideas and experimental proposals that have been presented. In fact, there are already several review papers and one very recent book (A. Bohm *et al.*, Springer 2003) where the diverse developments of the subject are summarized and reviewed. These articles and the monograph are listed and briefly described in the present book. What distinguishes the present book from the other review texts is that it treats both the quantum and classical geometric phases from a unified (fibre bundle connection) point of view at an advanced textbook level. In this respect, it is very helpful that the book contains a section on the geometric formulation of quantum mechanics and symplectic structure of quantum dynamics which is presented in *Chapter 5*. The book also discusses some very recent developments and applications, in particular, it presents a brief overview of the quantum entangle-

ment and of the holonomic quantum computation.

The exposition begins with a concise pedagogical introduction to the basic differential geometry notions (manifolds, differential forms, fibre bundles, connections, characteristic classes, homotopy theory) and of Lie groups and algebras. It continues with the examination of adiabatic phases in quantum and classical mechanics (Berry's phase, non-abelian Wilczek-Zee phase, Hannay's angle, Aharonov-Anandan and Pancharatnam phases) and concludes with *Chapter 6*, where the geometric phases are shown "in action" on several important physical examples, such as the Chiao-Tomita-Wu phase and Rytov's law in polarization optics, Aharonov-Bohm, Aharonov-Casher and quantum Hall effects, Mead quadrupole system and an improved Born-Oppenheimer approximation in molecular physics, and holonomic quantum computation (briefly). Every chapter contains illustrative problems and an annotated list of references for further reading. The list of references (at the and of the book) is fairly large – it contains 268 items, including 60 textbooks and monographs, and 6 recent E-prints.

The book is well written and I enjoyed reading it. The choice of topics presented and their mathematical description and physical explanation are well balanced and appropriate.

The book is aimed mainly at graduate students in mathematics and theoretical physics. It would, however, also be useful to (young and mature) researchers in different fields of theoretical physics and chemistry who want to familiarize themselves with the modern differential geometry methods and techniques, and to mathematicians, who wish to learn about a class of applications of the above methods in classical and quantum theory.

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