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INDEFINITE EISENSTEIN LATTICES: A MODERN BALL-RENDEVOUS WITH POINCARÉ, PICARD, HECKE, SHIMURA, MUMFORD, DELIGNE AND HIRZEBRUCH*

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Abstract. In [21] we have counted indefinite metrics (two-dimensional, integrally defined, over Gauss numbers) with a fixed norm (discriminant). We would like to call them also *indefinite class numbers*. In this article we change from Gauss to Eisenstein numbers. We have to work on the complex two-dimensional unit ball, an Eisenstein lattice on it and the quotient surface. It turns out that the compactified quotient is the complex plane \mathbb{P}^2 . In the first part we present a new proof of this fact. In the second part we construct explicitly a Heegner series with the help of Legendre-symbol coefficients. They can be interpreted as "indefinite class numbers" we look for. Geometrically they appear also as number of plane curves with (normed) Eisenstein disc uniformization.

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Contents

| 1 | Preface The Eisenstein Congruence Surface | | | 2 5 |
|----------------------------------|----------------------------------------------|------------------------|--------------------------------------------------------|--------|
| 2 | | | | |
| | 2.1 | Introdu | uction | 5 |
| | | 2.1.1. | Preview: Picard Modular Surfaces of Eisenstein Numbers | 8 |
| | 2.2 | Unimo | dular Sublattices | 10 |
| 2.3 C | | Counti | Counting Special Points | |
| | | 2.3.1. | Cusp Points on $\widehat{\Gamma \setminus \mathbb{B}}$ | 12 |
| | | 2.3.2. | \mathbb{B} -points of Maximal Negative Norm -1 | 13 |
| | | 2.3.3. | Norms of K-Discs and Their Quotient Curves | 14 |
| | | 2.3.4. | Stabilizing Subgroups | 15 |
| 2.4 All Elements of Finite Order | | ements of Finite Order | 17 | |

^{*}Dedicated to the memory of Professor Vasil V. Tsanov 1948-2017.