

JOURNAL OF

Geometry and Symmetry in Physics

ISSN 1312-5192

## **BI-HAMILTONIAN STRUCTURES ON THE TANGENT BUNDLE TO A POISSON MANIFOLD**

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Communicated by Alexandar B. Yanovski

**Abstract.** In the case when M is equipped with a bi-Hamiltonian structure  $(M, \pi_1, \pi_2)$  we show how to construct family of Poisson structures on the tangent bundle TM to a Poisson manifold. Moreover we present how to find Casimir functions for those structures and we discuss some particular examples.

*MSC*: 53D17, 37K10 *Keywords*: bi-Hamiltonian structure, Casimir function, Lagrange top, Lie algebra, Lie algebroid, linear Poisson structure, tangent lift of Poisson structure

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## 1. Introduction

The theories of Poisson and bi-Hamiltonian manifolds are one of important tools of the theory of integrable systems, see [1, 2, 10, 18, 24, 29]. The theory of Lie algebroids is another useful tool (see e.g. [3, 4, 9, 12, 13, 16, 31]) There are links between Poisson manifolds and Lie algebroids. It is well known that the total space of the dual bundle of a Lie algebroid has a canonical Poisson structure and there exists the canonical algebroid bracket of differential forms  $A = T^*M$ , where