

# HOLOMORPHIC PATH INTEGRALS IN TANGENT SPACE FOR FLAT MANIFOLDS

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**Abstract.** Here we study the quantum evolution in a flat Riemannian manifold. The holomorphic functions are defined on the cotangent bundle of this manifold. We construct Hilbert spaces of holomorphic functions in which the scalar product is defined using the exponential map. The quantum evolution is proposed by means of an infinitesimal propagator and the holomorphic Feynman integral is developed via the exponential map. The integration corresponding to each step of the Feynman integral is performed in the tangent space. Moreover, the method proposed in this paper naturally takes into account paths that must be included in the development of the corresponding Feynman integral. In the last section we apply our quantization method to the case when the configuration space is a space form, and we show the evolution operator as a composition of infinitesimal evolution operators.

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*Keywords:* Quantization, reproducing kernel Hilbert spaces, space forms

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