JGSP **64** (2022) 1–8

JOURNAL OF
Geometry and Symmetry in Physics
ISSN 1312-5192

GEOMETRIC PREQUANTIZATION OF THE SEIBERG-WITTEN MODULI SPACE ON THE PRODUCT OF A RIEMANN SURFACE

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Communicated by Gregory L. Naber

We show the existence of a symplectic structure on the moduli space of the Seiberg-Witten equations on $\Sigma \times \Sigma$ where Σ is a compact Riemann surface. To prequantize the moduli space, we construct a Quillen-type determinant line bundle on it and show its curvature is proportional to the symplectic form.

MSC: 14D21, 53-XX, 81-XX
Keywords: Geometric quantization, moduli spaces, Quillen bundle, Seiberg-Witten equations

1. Introduction

Let $\mathcal{M}$ be a symplectic manifold with an integral symplectic form Ω. Geometric prequantization is a construction of a line bundle with a Hermitian metric and a connection such the curvature is proportional to the symplectic form Ω. Moduli spaces of solutions of equations in gauge theory often have symplectic structure and we have the Quillen-type determinant bundles as prequantum line bundles. By Quillen-type bundles we mean either a Quillen bundle with a modified metric or pullback of a Quillen bundle with a modified metric. Examples include the vortex moduli space [1, 6], the Hitchin moduli space [2, 8], dimensionally reduced generalized Seiberg-Witten moduli space [4], etc. In each of these cases the equations are defined on fields on a Riemann surface. In each of these cases, we construct Quillen-type prequantum bundles which seem natural. The moduli space of the anti-self-duality equations also has a similar treatment [5], even though the fields are defined on a complex four manifold. An analogous technique is used in [7] for quantizing vortex moduli space for a complex Kähler four manifold. For an exposition on ASD equations, Mathai-Quillen formalism and the Witten conjecture see [9] and [10].

In this paper we focus on the moduli space of the Seiberg-Witten equations on the four manifold $\Sigma \times \Sigma$. We show that the moduli space carries symplectic structure using a moment map construction. Since we are on a four manifold, the classical