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A NOTE ON EKELAND'S VARIATIONAL PRINCIPLE AND CARISTI'S FIXED POINT THEOREM

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In this short note we present a new proof of Ekeland's variational principle and Caristi's fixed point theorem using a recently proved constrained variational principle in completely regular topological spaces.

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1. Introduction

A bounded below function attains its infimum if we suppose some type of continuity and compactness. However in many situations this is not the case. For example, functions defined in Hilbert spaces which are continuous in the norm topology but not in the weak topology. The question which is of interest is under what conditions we can perturb the function with another one from a prescribed class of functions so that perturbation has a solution. Such problems are called variational principles in optimization. Ekeland's variational principle [2], is one of the most powerful tools to solve problems in optimization. The principle has been shown to be equivalent to completeness of metric spaces. It leads to a quick proof of some fixed point theorems and is equivalent to Caristi's fixed point theorem. A new proof of these two equivalent theorems is provided using a constrained variational principle.

2. Preliminaries

First we present a constrained variational principle in completely regular topological spaces. Let Z be a completely regular topological space and $f\colon Z \to [-\infty, \infty]$ is a real-valued function. The set $\mathrm{dom}(f)$ is the domain of f and consists of all points in Z at which f is finite. The function f is called proper if its domain