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DEFORMAION QUANTIZATION WITH SEPARATION OF VARIABLES FOR COMPLEX TWO-DIMENSIONAL LOCALLY SYMMETRIC KÄHLER MANIFOLD

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A construction methods of noncommutative locally symmetric Kähler manifolds via a deformation quantization with separation of variables was proposed by Sako-Suzuki-Umetsu and Hara-Sako. This construction gives the recurrence relations to determine the star product. These recurrence relations were solved for the case of the arbitrary one-dimensional ones, N-dimensional complex space, complex projective space and complex hyperbolic space. For any two-dimensional case, authors found the solution of the recurrence relations. In this paper, we review the solution and make the star product for two-dimensional complex projective space as a concrete example of this solution.

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1. Introduction

Deformation quantization is one of the quantization methods to construct noncommutative differentiable manifolds, proposed by Bayen *et al* [2]. A deformation quantization for a Poisson manifold M is defined by the pair of the ring of formal power series over $C^\infty(M)$ and a star product $*$ on $C^\infty(M)[\![\hbar]\!]$. Here, $C^\infty(M)[\![\hbar]\!] := \{f;\ f = \sum_k f_k \hbar^k,\ f_k \in C^\infty(M)\}$, where $\hbar$ is a formal parameter. A star product $*$ is a product denoted by

$$f * g = \sum_{k=0}^{\infty} C_k(f,g)\hbar^k$$

for any $f, g \in C^\infty(M)$, which satisfies associativity, $C_k(\cdot,\cdot)$ is bi-differential operators, $C_0(f,g) = fg$, $C_1(f,g) - C_1(g,f) = \{f,g\}$, where $\{\cdot,\cdot\}\colon C^\infty(M) \times C^\infty(M) \to C^\infty(M)$ is a Poisson bracket, and $f * 1 = 1 * f = f$. The construction methods of the deformation quantizations for symplectic manifolds by de