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SOME EXACT SOLUTIONS OF ABC AND MARTÍNEZ ALONSO-SHABAT EQUATIONS

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Certain exact solutions (the so called functionally-invariant solutions) of the ABC equation and some two Martínez Alonso-Shabat equations, have been obtained by using the so-called structural decomposition method. Some of these solutions are localized.

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1. Introduction

In this paper we consider the equations

1. The so-called ABC equation [7]

$$Aq_tq_{xy} + Bq_xq_{ty} + Cq_yq_{tx} = 0 \tag{1}$$

where $q_x \equiv \partial q/\partial x$, etc., and we assume $A + B + C \neq 0$. This equation describes three-dimensional Veronese webs [4].

2. The so-called Martínez Alonso-Shabat equation [10]

$$u_{ty} = u_z u_{xy} - u_y u_{xz}. \tag{2}$$

3. The so-called modified Martínez Alonso-Shabat equation [10]

$$q_yq_{xz} + \lambda q_xq_{ty} - (q_z + \lambda q_t)q_{xy} = 0. \tag{3}$$

Some exact solutions (functionally-invariant solutions) of the $(2+1)$-dimensional modified Veronese equation, were found in [8]. In [2], the integrability of the equation (1) had been shown, in the case $A + B + C \neq 0$. In [4] a correspondence between Veronese webs and three-dimensional Lorentzian Einstein-Weyl