SPECTRAL GEOMETRY OF UNDULOIDS

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This paper examines the eigenvalues and eigenfunctions of the Laplace operator associated with a set of mathematically defined surfaces which can be produced experimentally by attaching two equally sized rings to opposite poles of a soap bubble and separating the rings. The shapes produced are called unduloids. These calculations show 1) for a range of ring sizes, as a function of ring separation, the first indexed eigenvalue has a minimum, pointing to an “optimum” shape, and 2) given the eigenfunctions in the form of their differential equations and a preference for symmetry, the underlying unduloid geometry may be deduced.

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