



EINSTEIN METRICS ON COMPACT SIMPLE LIE GROUPS

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A Riemannian manifold is called Einstein if it has constant Ricci curvature. Here we present an overview of recent results about left invariant Einstein metrics on compact simple Lie groups, which are not naturally reductive. We also explain methods, techniques and tools.

MSC: 13P10, 53C25, 53C30, 65H10, 68W30

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