LIE ALGEBRAS, EIGENVALUE PROBLEMS AND LEFT-SYMMETRIC ALGEBRAS

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In this paper we will show a relation between class of Lie algebras given by eigenvalue problem and left-symmetric algebras. We also define multiplication on linear spaces originating from the eigenvalue problem.

1. Introduction

Our goal is to show that eigenvalue problem is connected, in natural way, with left-symmetric algebras and their generalizations and therefore it is closely related with Lie algebras.

Recently in paper [4] it was shown that there exists a close relation between classical objects such as Lie algebras and eigenvalue problems. Every eigenvalue problem gives Lie bracket and Lie algebra received in such a way is strongly connected with Lie algebra of the generalized $ax + b$-group. The collection of eigenvalue problems can give any finite dimensional Lie algebra. On the other hand every Lie algebra gives some collection of eigenvalue problems. It should be added that it is not one-to-one correspondence. This concept was widely discussed in [3] and [6].

The aim of this paper is to show connection with left-symmetric algebras. This notion generalizes notion of associative algebra by allowing possibility of the associativity not to be fulfilled. It is well known fact that any left-symmetric algebra is pre-Lie algebra. We will show that Lie algebras given by one eigenvalue problem are always also left-symmetric Lie algebras. More interesting situation appears in the case of more than one eigenvalue problem (essentially independent). It will be shown that the eigenvalue problem provides multiplication in this Lie algebra. We will demonstrate that in detail in case of three dimensional Lie algebras.