



SYMMETRIES AND CONSERVATION LAWS OF A PERTURBED QUARTIC INTERACTION SYSTEM DISCUSSED BY FRACTIONAL φ^4 -EQUATION

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The aim of the paper is to provide a conceptual theoretical framework based on symmetry analysis of differential equations in order to deal on the invariance properties of a physical perturbed fractional PDE. Also with the help of the concept of non-linear self-adjointness and the definition of conserved vector, approximate conservation laws are obtained without approximate Lagrangians by using their approximate symmetries. In order to apply the presented theory, we implement Lie symmetry analysis and concept of non-linear self-adjointness to reach the exact solutions, perturbed and unperturbed symmetries and conservation laws of converted equation. This indicates a need to introduce and extend the methodology of symmetry analysis of differential equations on the system of the kind of both fractional and perturbed form.

MSC: 34K37, 76M60

Keywords: Approximate conservation law, fractional calculus, Lie symmetry, optimal system, similarity solutions

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