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ALEXANDAR B. YANOVSKI

IN MEMORIAM

The Bulgarian mathematician Professor Alexandar Borissov Yanovski passed away on October 27, 2023. The following is a short resume of his life and achievements as a teacher, scientist and person based on the reminiscences of his colleagues, co-authors and friends.

Introduction

Alexandar Borissov Yanovski was born on 16 April 1953 in Sofia, Bulgaria. He graduated from the Faculty of Physics of the St. Kliment Ohridski State University in Sofia in 1976. In 1978 he successfully defended his MSc thesis under the supervision of Prof. I. Todorov at Institute for Nuclear Research and Nuclear Energy, Sofia, see also [31].

After graduation, A. Yanovski started working at the same Faculty. He joined the research group of Prof. N. Nikolov, studying electromagnetic waves in plasma. This results in his first series of publications [1–6] on the theory of propagation of electromagnetic waves in plasma and specifically in gas plasma wave-guides.

From 1982 to 1988 Alexandar became a Research Associate and a Senior Research Fellow at the Joint Institute for Nuclear Research (JINR) in Dubna (SSSR). He worked at the JINR Laboratory of Computing Techniques and Automation (LCTA) in the research group of Prof. V. Makhankov. At this period, his main scientific interests shifted to integrable systems. His second series of works with Dr. A. Shvachka were devoted to the Wahlquist-Estabrook prolongation method and its application to nonlinear evolution equations [7–9] and on Magri's theorem for complete integrability (together with B. Florko) [10]. Meanwhile, he and his family met in Dubna the families of Bordag (Ljudmila and Michael), Igor Barashenkov and Vladimir Gerdjikov – these contacts significantly influenced his later scientific career. In 1987 Alexandar Yanovski defended his PhD thesis: "*Gauge-Covariant Approach to the Theory of the Generating Operators for Nonlinear Soliton Equations*" under the supervision of Prof. V. Makhankov and Dr. V. Gerdjikov.

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Prof. Alexandar Yanovski in his office at University of Cape Town.

After returning from Dubna, in 1988 Alexandar Yanovski was appointed at the Faculty of Mathematics and Informatics of Sofia University as an Lecturer, becoming Associate Professor at the same faculty in 1995. His research interest shifted to spectral aspects of Lax operators [20], differential-geometric methods and their applications to the theory of integrable systems, including the theory of bi-Hamiltonian systems, the geometric theory of (generating) recursion operators and the related tensor fields on manifolds. In 1993 he launched an advanced undergraduate elective course "Differential Geometry and Analytical Mechanics", which quickly gained popularity among the students in the Faculty of Mathematics and Informatics in the second-half of 90's.

From 1994 to 1997 Alexandar Yanovski participated in a scientific collaboration with Leipzig University, Germany, organized by Ljudmila Bordag, dedicated to integrable systems and he has visited Leipzig number of times for several weeks at a time.

In 1999 Alexandar Yanovski became a visiting professor at the Federal University of Sergipe, Aracaju, Brazil. During that time he won three selection processes for permanent position in states universities of Aracaju, Curitiba and Sao Paulo.

In 2003, Prof. I. Barashenkov invited him to apply for permanent position at the Maths Department of University of Cape Town (UCT) and he was appointed as an Associate Professor at the University of Cape Town, South Africa, until his retirement at the end of 2018. At the University of Cape Town he delivered various mathematical courses, including Mathematical Analysis, Complex Analysis and Differential Geometry as well as various elective advanced-undergraduate and graduate courses.

During this time he wrote a number of single-authored papers [28–33, 36–38, 40, 42,43,48–52,55,60]. He also continued his active collaboration with Prof. V. Gerdjikov and Prof. Gaetano Vilasi from Salerno University, Italy. One of the important results of this collaboration was the monograph [34], several joint papers with Prof. G. Vilasi [39, 41, 54] and later, with Dr. T. Valchev [56–59, 61].

He also made significant contribution to the spectral theory of Lax operators and the associated generating (recursion) operators, the generalized Fourier decompositions, that linearize the nonlinear evolution equations (NLEE) and for the geometric aspects of the theory of integrable systems.

His results, along with those of his colleagues and co-authors V. Gerdjikov, D. Kaup, N. Kostov, G. Grahovski, T. Valchev, G. Vilasi, R. Dandolov, R. Ivanov, R. Dandoloff and others were awarded the 2007 Prize for Best Work in Theoretical Physics by the Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences. The award was given for important contribution in the area of completely integrable equations. This research resulted in a Springer monograph [34] and a large number of publications with hundreds of citations.

In addition, besides high quality research, Prof. Yanovski also demonstrated high level of pedagogical abilities. During his work at Sofia University, Sergipe University and Cape Town University he had given courses on numerous topics, including Classical Electrodynamics, Symplectic Geometry and Analytical Mechanics, Differential Geometric Methods of Analytical Mechanics, Geometric Approach to the Soliton Equations, Differential Geometry and Hamiltonian Mechanics, Ordinary Differential equations, Methods of Mathematical Physics and others.

Last but not least, Prof. Yanovski was very passionate in promoting and disseminating Science. Let us mention especially that he was a valuable member of the Editorial Board of Journal of Geometry and Symmetry in Physics who with his deep knowledge of modern Mathematics and Theoretical Physics and broad research interests helped in establishing the international reputation of the journal and made it an attractive place for researchers in the field to submit high quality works.

In the next section we sketch Alexandar Yanovski's main scientific achievements.

1. Scientific Achievements and Main Results

As we mentioned in the Introduction, the main results of Alexandar Yanovski are related to completely integrable systems of PDEs[1], which allow Lax representation $[L, M] = 0$. Generically speaking, the linear operators L and M are ordinary differential operators of the form:

$$\begin{aligned} L\psi &\equiv \mathrm{i}\frac{\partial \psi}{\partial x} + U(x,t,\lambda)\psi(x,t,\lambda) = 0, \qquad U(x,t,\lambda) = \sum_{s=0}^{N} U_s(x,t)\lambda^{N-s} \\ M\psi &\equiv \mathrm{i}\frac{\partial \psi}{\partial t} + V(x,t,\lambda)\psi(x,t,\lambda) = 0, \qquad V(x,t,\lambda) = \sum_{s=0}^{P} V_s(x,t)\lambda^{P-s}. \end{aligned} \tag{1}$$

Typically, the dependence of L on the spectral parameter λ is polynomial of order $N = 1$ or two. In addition, the gauge of L is fixed by requesting that U_0 is constant diagonal matrix. For the second Lax operator M, we may have $P = N$ or $P = 2N$. We start with this generic form of the Lax pair because the gauge covariant approach may be directly generalized for any polynomial dependence of the Lax pair. Here we also mention that the Lax representation $[L, M] = 0$ must hold identically with respect to $\lambda \in \mathbb{C}$.

1.1. The Gauge Covariant Approach to Integrable NLEE

The gauge covariant approach was formulated first for the simplest nontrivial Lax pair, known as the Zakharov-Shabat system [72]

$$\begin{aligned} L_{\mathrm{ZSh}}\psi_{\mathrm{ZSh}} &\equiv \mathrm{i}\frac{\partial \psi_{\mathrm{ZSh}}}{\partial x} + (Q(x,t) - \lambda\sigma_3)\psi_{\mathrm{ZSh}}(x,t,\lambda) = 0 \\ Q(x,t) &= \begin{pmatrix} 0 & q(x,t) \\ \epsilon q^*(x,t) & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \epsilon = \pm 1 \end{aligned} \tag{2}$$

and its gauge equivalent

$$\begin{aligned} L_{\mathrm{HF}}\psi_{\mathrm{HF}} &\equiv \mathrm{i}\frac{\partial \psi_{\mathrm{HF}}}{\partial x} - \lambda S(x,t)\psi_{\mathrm{HF}}(x,t,\lambda) = 0 \\ S(x,t) &= g^{-1}(x,t)\sigma_3 g(x,t), \qquad g(x,t) = \psi_{\mathrm{ZSh}}(x,t,\lambda = 0). \end{aligned} \tag{3}$$

The famous Zakharov-Shabat system L_{ZSh} generates the nonlinear Schhrödinger (NLS) equation

$$\mathrm{i}\frac{\partial q}{\partial t} + \frac{\partial^2 q}{\partial x^2} + 2\epsilon|q|^2 q(x,t) = 0 \tag{4}$$

[1] For basic notations and background material about classical integrability, we refer to [34].

while its gauge equivalent NLEE is the Heisenberg ferromagnet (HF) equation

$$i\frac{\partial S}{\partial t} - [S, S_{xx}] = 0. \tag{5}$$

The gauge covariant approach allows one to see that the spectral properties of the operators $L_{\rm ZSh}$ and $L_{\rm HF}$ are equivalent. This allows one to prove that the NLEEs (4) and (5) are gauge equivalent. It allows us also to construct the spectral decompositions of the relevant generating (recursion) operators $\Lambda_{\rm ZSh}$ and Λ_{HF} [11–13, 15]. More details and explanations can be found in Part I of the monograph [34].

1.2. Spectral Properties of the Lax and Recursion Operators

The inverse scattering method (ISM) can be applied to any operator L for which the inverse scattering problem allows unique solution. For $L_{\rm ZSh}$ this is guaranteed by the Gelfand-Levitan-Marchenko (GLM) equation which ensures the analyticity properties of the Jost solutions.

The generalized Zakharov-Shabat systems, as discovered by Zakharov and Manakov [71]:

$$L_{\rm gZS} = i\frac{\partial \psi_{\rm gZS}}{\partial x} + (Q(x,t) - \lambda J)\psi_{\rm gZS}(x,t,\lambda) = 0 \tag{6}$$

allows one to solve other important NLEE, like the N-wave equations, having applications in Nonlinear Optics. Here $Q(x,t)$ is an element of some simple Lie algebra $\mathfrak{g}$, and J is an element of its Cartan subalgebra $\mathfrak{h}$. We assume that J has real eigenvalues a_j that are ordered by $a_1 > a_2 > \cdots > a_n$. For such systems, GLM-type equation can not be derived. The solution of the inverse scattering problem for $L_{\rm gZS}$ was found by Shabat [70], who proposed effective construction of the fundamental analytic solutions $\chi^{\pm}(x,t,\lambda)$ (FAS) of $L_{\rm gZS}$. They are related to the Jost solutions of $L_{\rm gZS}$ by

$$\chi^{\pm}(x,t,\lambda) = \phi(x,t,\lambda)S^{\pm}(t,\lambda) = \psi(x,t,\lambda)T^{\mp}(t,\lambda)D^{\pm}(\lambda) \tag{7}$$

where $S^{\pm}(t,\lambda)$, $D^{\pm}(\lambda)$ and $T^{\mp}(t,\lambda)$ are the factor in the Gauss decompositions of the scattering matrix $T(t,\lambda)$, see, e.g., [64, 69]

$$T(t,\lambda) = T^{-}(t,\lambda)D^{+}(\lambda)S^{+,-1}(t,\lambda) = T^{+}(t,\lambda)D^{-}(\lambda)S^{-,-1}(t,\lambda). \tag{8}$$

Here $T^{+}(t,\lambda)$ and $S^{+}(t,\lambda)$ are upper-triangular matrices whose diagonal elements are all equal to one and $T^{-}(t,\lambda)$, $S^{-}(t,\lambda)$ are lower-triangular matrices whose diagonal elements are all also equal to one. Respectively $D^{+}(\lambda)$ and $D^{-}(\lambda)$ are diagonal matrices which allow analytic extensions for ${\rm im}\lambda > 0$ and ${\rm im}\lambda < 0$. Note that if $T(t,\lambda)$ is an element of the simple Lie group $\mathcal{G}$, then the matrix elements of all Gauss factors in (8) can be evaluated explicitly as ratios of minors of $T(t,\lambda)$, see [64]. In particular, the matrix elements of $D^{\pm}(\lambda)$ are ratios of the principle minors of $T(t,\lambda)$ and provide the generating functionals of the integrals of motion for the NLEE.

The next class of Lax operators were proposed by Beals and Coifman [63] (BC). Their form is similar to the one of L_{gZS}

$$L_{\mathrm{BC}} = \mathrm{i}\frac{\partial \psi_{\mathrm{BC}}}{\partial x} + (Q(x,t) - \lambda J)\psi_{\mathrm{BC}}(x,t,\lambda) = 0 \tag{9}$$

where $Q(x,t)$ and J belong to the algebra $sl(n)$ and $J = \mathrm{diag}(c_1, c_2, \ldots, c_n)$ has complex eigenvalues. The analyticity properties of L_{BC} are substantially different as compare to L_{gZS}. First we need to find the regions of analyticity which are determined by the conditions:

$$\mathrm{im}\lambda(c_k - c_j) = 0. \tag{10}$$

The solution to eqs. (10) is a set of straight line $\arg\lambda = \beta_s$ passing through the origin; the angles β_s, $s = 1, 2, \ldots, P$ are determined by the eigenvalues c_j of J. These lines split the complex λ plane into $2P$ sectors Ω_s. In each sector one can construct FAS of L_{BC}. Alexandar came up with the idea to generalize these results to the case when $Q(x,t) \in \mathfrak{g}$ and $J = \sum_{j=1}^{r} c_j H_{e_j} \in \mathfrak{h}$, where H_{e_j} are the Cartan generators of $\mathfrak{g}$ of rank r. In this case the conditions (10) change into

$$\mathrm{im}\lambda\alpha(J) = 0, \qquad \alpha \in \Delta_{\mathfrak{g}} \tag{11}$$

where α is a root of the algebra $\mathfrak{g}$. If the algebra $\mathfrak{g} \simeq \mathfrak{sl}(n)$, then the root system $\Delta_{\mathfrak{g}} \equiv \{e_j - e_k\}$ and condition (11) coincides with (10). This generalization was published in [20]. Later on these results were used in the analysis of spectra of Lax operators subject to $\mathbb{Z}_h$ and $\mathbb{D}_h$ reduction groups, see [67]. Such operators generate the hierarchies of NLEE containing the two–dimensional Toda field theories [67, 68]. In [43–45], along with the description of the spectral properties of such Lax operators, the completeness relations for their 'squared solutions' were also derived. These expansions map the variation of the potential $\delta Q(x,t)$ onto the variations of the action-angle variables of the two-dimensional Toda field theories.

Completeness of the "squared solutions". We first introduce the so-called "squared solutions" of the Lax operator L related to a given simple Lie algebra $\mathfrak{g}$ with fixed $J \in \mathfrak{h}$. Using J, we can split the set of positive roots Δ^+ of $\mathfrak{g}$ into two subsets $\Delta^+ = \Delta_0^+ \cup \Delta_1^+$

$$\Delta_0^+ \equiv \{\alpha \in \Delta^+,\ \alpha(J) = 0\}, \qquad \Delta_1^+ \equiv \{\beta \in \Delta^+,\ \beta(J) > 0\}. \tag{12}$$

We can also introduce the projector $\pi_J = \mathrm{ad}_J^{-1}\mathrm{ad}_J$, which acts on a generic element $X = \sum_{j=1}^{r} x_j H_{e_j} + \sum_{\alpha\in\Delta^+\cup\Delta^-} x_\alpha E_\alpha$ by

$$\pi_J X = \sum_{\alpha\in\Delta_1^+} (x_\alpha E_\alpha + x_{-\alpha} E_{-\alpha}). \tag{13}$$

The complete set of 'squared solutions' for the generalized Zakharov-Shabat system (6) consists of

$$e^{\pm}_{\pm\beta} = \pi_J \chi^{\pm}(x,\lambda) E_{\pm\beta} \chi^{\pm,-1}(x,\lambda) \tag{14}$$

where $\chi^{\pm}(x,\lambda)$ are the FAS of L_{gZS}. Using the Wronskian relations, one can prove that these 'squared solutions' map the potential $Q(x,t)$ into the scattering data, see, e.g., [64, 65]. Thus, one finds that the interpretation of the ISM as generalized Fourier transform [62] in fact holds true for a much larger class of Lax operators, including L_{gZS} [64], L_{BC} [44, 47].

The 'squared solutions' are known also as generalized exponentials that are used to demonstrate the fact that the ISM has the meaning of generalized Fourier transform. They are used also to display that the integrable NLEE allow hierarchies of Hamiltonian structures. Here Alexandar had important contributions, see [27–29, 35], as well as the monograph [34]. At the same time the 'squared solutions' are eigenfunctions of the recursion operator, which explains why the set of all NLEE integrable with L_{gZS} have the form

$$\mathrm{ad}_J^{-1} \frac{\partial Q}{\partial t} + f(\Lambda) Q(x,t) = 0 \tag{15}$$

where Λ is the recursion operator and $f(\lambda)$ is the dispersion law of the NLEE, see [64, 65].

1.3. The Chiral O(3)-Field Equations and Landau-Lifshitz Equations and Their Hierarchies

From 1994 to 1997 Alexandar Yanovski participated in research projects organized by Ljudmila Bordag at Leipzig University, where she worked as an Associate Professor at that time. The projects were funded by German Academic Exchange Service (DAAD) and the Center for Natural Sciences and Technology (NTZ) at Leipzig University. Supported by this project, Alexandar Yanovski came to Leipzig for several weeks at a time to work with L. Bordag on problems of a common interest and to deliver a comprehensive short course of lectures for advanced students and staff members on current topics in integrable systems. The lectures were well accepted by students and published as preprints of Leipzig University [22, 24].

It was Alexandar Yanovski's idea to look for new Lax pairs for chiral O(3)-field equations and the Landau-Lifshitz equation. At that time it was a novel idea, since well-known Lax pairs for each of these equations do exist and were ready to use! However, the known Lax pairs were not very convenient. They do depend on the spectral parameter λ through elliptic functions. The idea was to use Lie algebras to obtain new Lax pairs having a polynomial dependence on the spectral parameter.

Firstly, it was possible to find completely new polynomial pairs in the form of 6×6-matrices with quadratic dependence on the spectral parameter λ. Using the isomorphism between algebras $\mathfrak{so}(3, \mathbb{R})$ and $\mathfrak{su}(2)$, one can also obtain the old Lax pair as a 2×2 matrix from the new one.

Secondly, using the Lie algebra isomorphisms between the complexification algebra $\mathfrak{so}(3, 3)$ and $\mathfrak{so}(6, \mathbb{C})$ as well as the ones between $\mathfrak{so}(6, \mathbb{C})$ and the algebra $\mathfrak{sl}(4, \mathbb{C})$, one can obtain a new polynomial Lax pair in the form of 4×4 matrices. These Lax pairs are much more convenient than the Lax presented by the larger matrices. It is worth noting that building a Lax pair is not a simple algorithmic procedure – it requires an intuition and a bit of a fortune to get a result. The results of this work are published in [21].

The second joint work [23], was a logical development and application of the results of [21]. Using the new Lax pairs one can build new polynomial hierarchies for the chiral O(3)-field equations and Landau-Lifshitz equations. The set of polynomial Lax pairs for the chiral O(3)-field equations and the corresponding hierarchy of equations were presented in this work for the first time. For the Landau-Lifshitz equations the hierarchy of soliton equations obtained via elliptic bundles was known before. It means that for these hierarchies it is possible to provide some comparison of the associated soliton equations. The first equations in both hierarchies did coincide identically. The second and third ones were equivalent to each other, but it was not possible to prove that all the soliton equations in both hierarchies are equivalent. The authors believe, that this is the case.

1.4. Geometric Aspects of the Recursion Operators

Geometric aspects of the recursion operators were another important subject in Prof. Alexandar Yanovski's expertise. They form the basis of the Hamiltonian hierarchies of the NLEE and also include Poisson manifolds that allow Schouten brackets. Alexandar was recently working with Gaetano Vilasi on the Nambu formulation (ternary Poisson brackets) of Landau-Lifchitz dynamics (see [34, 39, 41, 54] and several single-authored ones [16–19, 22, 37, 40, 53]).

Here we should also mention his results on the locality of conservation laws [14, 46].

1.5. On Lax Operators in Pole Gauge

In [57], the auxiliary spectral problem introduced by Gerdjikov, Mikhailov and Valchev [66] and its pseudo-Hermitian modification are extensively studied in the case of the Lie algebra $\mathfrak{sl}(3,\mathbb{C})$. The integrable hierarchies associated with both spectral problems are described by using generating operators. Based on the notion of gauge equivalent auxiliary linear systems, the authors derive completeness relations for the eigenfunctions of the generating operators that are related to the Lie algebra $\mathfrak{sl}(3,\mathbb{C})$. This allows one to obtain general expansion formulae for both spectral problems with arbitrary constant boundary conditions imposed on the potential. More general expansions over the eigenfunctions of those generating operators that also take into account the discrete part of the spectrum of the scattering operator have been found in [59]. It is shown how these expansions change under the presence of discrete symmetries of the functions being expanded.

The paper [58] is a continuation of [57]. The authors demonstrate how one can apply the dressing method to linear bundle Lax pairs in pole gauge that are connected to $\mathfrak{sl}(3,\mathbb{C})$. This approach is illustrated on the Lax pair of a nonlinear evolution equation that generalizes Heisenberg ferromagnet equation. This lets one construct two types of explicit solutions over constant background: soliton solutions and quasi-rational solutions. Thus, an essential difference between the Hermitian and pseudo-Hermitian reduction becomes obvious – quasi-rational solutions exist in the latter case only. Both types of solutions are not traveling waves. These results are extended in [61] for a nonlinear evolution equation whose Lax representation is related to symmetric spaces of the type $\mathrm{SU}(n+1)/\mathrm{S}(\mathrm{U}(1)\times\mathrm{U}(n))$.

The report [56] extends some of the results published in [57]. A pseudo-Hermitian reduction of a new matrix S-integrable generalization of the classical Heisenberg ferromagnet equation related to the symmetric spaces $\mathrm{SU}(m+n)/\mathrm{S}(\mathrm{U}(m)\times\mathrm{U}(n))$ is introduced. Generating operators to describe the integrable hierarchy of that matrix equation are derived as well.

2. Memories of Colleagues and Friends

Prof. Ljudmila Bordag: We, my husband Michael and I, have known Alexandar and his lovely family since February 1983. Our family with two children – our four years old son Stefan and eight month old daughter Natalie, had just arrived in Dubna to work at JINR. We were delegated from the GDR. My husband Michael started work at Laboratory of Theoretical Physics (LTP) and I was busy with our young daughter waiting for a place in kindergarten, to start work at LCTA at the same Institute. We were new in Dubna and knew few people. While playing in the

Alexandar with his daughter Natalia in Dubna 1986 (left panel) and in Leipzig 1994 (right panel) with Stefan Bordag, Mikhail Babich, and Ljudmila Bordag.

sandbox behind our house, our daughter chose another small girl to play. I asked her name, it turned out to be the same name as my daughter's name – Natalia. Then we chatted with her mother, Iliana. Their family had also just arrived, but from Bulgaria. After an hour we went home and we realized that we were living in the same building, the same entrance, just on different floors.

Few weeks later I started to work in LCTA and got a workspace in a room with some young people. We worked in Prof. V. Makhankov small research group. One of our group members was from Bulgaria and was working on similar problems as me: inverse scattering problems, and connections of integrable systems with Painlevé equations. It was Alexandar. After few days I realized that he was the father of the little Natalia, whom our daughter chose as a friend. Since that time our families have been friends. During our four years in Dubna, we helped each other solving everyday problems and discussed scientific and philosophical issues. Alexandar had a very deep knowledge in many areas of Mathematics, Physics, and cultural life. He had a sharp and original view on many political and social phenomena. Thanks to his sense of humor and profound ideas, he was always a very interesting interlocutor in scientific discussions as well as in everyday life. Also, he was a very good chess player and won many times against my husband.

In 1987 we left Dubna and returned to the GDR. Both my husband and I continued working at the Leipzig University, I – at the Faculty of Mathematics and my husband – at the Faculty of Physics. The next two years were extremely difficult for our family because of health problems and the political crisis in the former GDR. The horrible nineties were difficult years in many countries of the

Alexandar Yanovski at Dubna in 1987.

former Soviet bloc, especially in former GDR for many families. But we managed to overcome these times and found a strength to continue working in the area of Science. I then worked on problems of algebro-geometric approach to nonlinear partial differential equations, theta-functional solutions of KdV-, KP-, sin-Gordon, sin-Laplace equation and other equations, Schottky uniformization of Riemann surfaces. In 1993 I successfully defended my habilitation thesis "Some Problems of Effectivization of Theta-Functional Solutions of Nonlinear Integrable Evolution Equation" at Leipzig University and began working on the theory of Lax pairs for nonlinear equations. At that time Alexandar was also working in this area and we joined our efforts. As a result, we co-authored two papers in 1995 and 1996 [21, 23]. At that time Alexandar was visiting Leipzig and giving some lectures in English for the staff and students of Leipzig University. His lectures were excellently prepared and despite dealing with very advanced material, he delivered them very clearly and elegantly. The lectures were published as preprints of Leipzig University [22, 24]. When I received my first Honor Professorship in 1997–1998, the Dorothea-Erxleben-Professor at Magdeburg University, Germany, we continued our scientific cooperation with Alexandar. He visited us in Magdeburg and some of his results from this period were published in [25, 26]. At that time I was working with Prof. Mikhail Babich on the theory of Painlevé equations.

Later, in 1999–2000, our paths crossed once again as his daughter lived in our house preparing to study at a German university. Since we were both looking for

Alexandar in Magdeburg with Stefan Bordag, Michael Bordag, Natalie Bordag (behind) and Vladimir Nikolaev (left panel) and in Leipzig 1998 (right panel).

permanent positions at that time, we traveled extensively to different countries. After 2003, when I got a permanent Professorship at Halmstad University in Sweden, I worked in the area of Financial Mathematics, using methods of Lie group analysis for nonlinear equations. Our families kept in sporadic contact with each other but our scientific themes developed in quite different directions so we did not collaborate on any more publications.

After our retirement, we thought about getting in touch with this nice family again, but then the COVID-19 pandemic started with the associated traveling restrictions. We are very saddened that Alexandar passed away so suddenly. We will always remember him and cherish his memory.

Iliana Ianovska: My husband was an extraordinary person in many ways. He had multiple talents. Because of his talents we were able to spend our life on three continents, in three different foreign countries: in the Soviet Union for six years, in Brazil for four and a half years and in South Africa for more than 20 years. In each of these countries he was held in high esteem as a scientist, a teacher, a colleague and a friend.

He worked on extremely abstract topics in theoretical physics and mathematics and at the same time he had deep knowledge of history, literature, politics, psychology and the arts.

He was able to explain complicated matters in an easy, light and understandable way. He was a storyteller with a fine sense of humor. It was always enchanting to listen to him while he told fairy tales to his grandsons, explained complicated mathematical topics to his students or discussed politics with his friends.

He was a good chess player, too.

He had a strong character and was able to stand up for his ideas and fight any injustice.

He spoke five languages fluently. He spoke Russian as a native speaker. While living in the Soviet Union people used to think that he was Russian. When we moved to Brazil, we did not know a single word of Portuguese. His contract with the State University of Sergipe allowed him to teach in English for only one year until he became familiar with Portuguese. Not long after he started lecturing there he realized that his students understood neither mathematics nor English. He really wanted to help them. That is why he made enormous efforts to learn the language and in six months he started lecturing in Portuguese without reading from supporting notes.

It was unthinkable for him to read from notes in front of his students while lecturing, like some of his colleagues did. He put time into preparing for every single lecture so that it flowed naturally and easily. He was like an actor on the stage while lecturing.

He was so good at doing so that, ironically, some students thought he did not prepare for his lectures at all. Others felt embarrassed because he could answer any question they might ask right away.

For him it was unacceptable to give lectures in shorts. Even with temperatures approaching 40 °C he always wore a shirt and trousers to his lectures.

On a professional level he always kept high standards in everything he did.

On a personal level he was loving husband and father. We were married for 42 years filled with love and tenderness.

He is deeply missed.

Prof. Haris Skokos[2]: I first met Alexandar Ianovsky in 2013 when I joined the Department of Mathematics and Applied Mathematics (MAM) at the University of Cape Town (UCT) where he was already working. Although I never collaborated with him on a research project, as our research interests were quite different, I had a very close relationship with him, in the department, as well as a friend. What impressed me particularly about Alexandar was his passion for his discipline, academia, and the well-being of his students. He was always trying to find ways to improve everyday life in the department, contributing innovative ideas about how we can meliorate our collaboration and interaction, as well as make it more productive and pleasant. At the same time, he was highly committed to advancing and promoting mathematics, not only as a researcher but, maybe most importantly, as

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With H. Skokos at Laangebaan (left panel) and at Stark-Conde, Stellenbosch (right panel) in 2015. In the right panel are also present Alexandar's wife Iliana Ianovska (far left) and Skokos' wife Eirini Intzoglou.

a university teacher. He was constantly pushing to deliver high-quality teaching, guidance, and mentoring to his students, aiming to transfer his deep love for mathematics to them. Additionally, I had the honor of knowing Alexandar on a personal level. He was a very warm and kind person, always able to tell you an interesting story, whether it was about mathematics or completely different topics based on his experiences of living in many different countries. One could always have a good laugh with him and I certainly enjoyed the evenings we spent together. Alexandar will be deeply missed by the people he worked and collaborated with, as well as his family and friends, and he will always be remembered as a very devoted and dedicated scientist.

3. Conclusions

Professor Alexandar Yanovski devoted his entire life to science, making significant contributions to the development of the theory of solitons. His scientific heritage includes more than 60 scientific publications in prestigious research journals and a monograph [34]. He also left a lucid trace among his colleagues and friends as a person with interests going beyond the borders of Mathematics and Physics. His natural and uniques sense of humor and his wide smile were making any interaction with him a remarkable and memorable experience.

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Publications by Alexandar B. Yanovski

[1] Nikolov N., Malinov P., Bourova M. and Yanovsky A., *On the Averaged Forces in the Limited Plasma in Crossed Fields*, C. R. Acad. Bulg. Sci. **34** (1981) 15–17.

[2] Nikolov N., Andreeva V., Yanovsky A. and Malinov P., *On the Theory of a Resonance Probe for Diagnostics of a Bounded Plasma in a Magnetic Field*, J. Phys. D: Appl. Phys. **15** (1982) 445–449.

[3] Nikolov N., Malinov P. and Yanovsky A., *On the Fields and Electromagnetic Energy in Bounded Plasma in a Magnetic Field*, Bulg. J. Phys. **9** (1982) 95–105.

[4] Nikolov N. and Yanovsky A., *Dispersion Relations for Non-Potential Waves in a Cylindrical Plasma Waveguide in a Finite External Magnetic Field*, Bulg. J. Phys. **10** (1983) 443–449.

[5] Nikolov N., Malinov P. and Yanovsky A., *On the Averaged Forces in Bounded Plasma in Parallel Electric and Magnetic Field*, Beitr. Plasmaphys. **23** (1983) 95–103.

[6] Nikolov N. and Yanovsky A., *On the Symmetrical Modes in a Cylindrical Non-isothermal Plasma Waveguide in a Finite External Magnetic Field*, Beitr. Plasmaphys. **24** (1984) 203–209.

[7] Shvachka A. and Yanovsky A., *The Wahlquist-Estabrook Method for the Investigation of the Nonlinear Models of Classical Field Theory*, JINR Comm. E-82-546, Dubna 1982.

[8] Shvachka A. and Yanovsky A., *The Wahlquist-Estabrook Method and Its Application to the Investigation of Nonlinear Evolution Equations. One-Dimensional Case*, JINR Comm. R5-82-239, Dubna 1982.

[9] Shvachka A. and Yanovsky A., *The Wahlquist-Estabrook Method and Its Application to the Investigation of Nonlinear Evolution Equations. Two-Dimensional Case*, JINR Comm. R5-82-242, Dubna 1982.

[10] Florko B. and Yanovski A., *On Magri's Theorem for Complete Integrability*, JINR Comm. E-83-831, Dubna 1983.

[11] Gerdjikov V. and Yanovski A., *Gauge Covariant Formulation of the Generating Operator for the Nonlinear Evolution Equations on Homogeneous Spaces*, JINR Comm. D-17-84-850, Dubna 1984.

[12] Gerdjikov V. and Yanovski A., *Gauge Covariant Formulation of the Generating Operator. I. The Zakharov-Shabat System*, Phys. Lett. A, **103** (1984) 232–236.

[13] Gerdjikov V. and Yanovski A., *Gauge Covariant Formulation of the Generating Operator. II. Systems on Homogeneous Spaces*, Phys. Lett. A, **110** (1985) 53–58.

[14] Gerdjikov V. and Yanovski A., *Generating Operator and Locality of the Conservation Laws. The Case of Zakharov-Shabat Linear Problem*, JINR Comm. R5-82-242, Dubna 1985.

[15] Gerdjikov V. and Yanovski A., *Gauge Covariant Theory of the Generating Operator. I*, Comm. Math. Phys. **103** (1986) 549–568.

[16] Yanovski A., *The Geometrical Meaning of the Generating Operators for the Generalized Zakharov-Shabat System in Pole Gauge*, JINR Comm. R5-87-334, Dubna 1987.

[17] Yanovski A., *The Geometrical Meaning of the Generating Operators for the Generalized Zakharov-Shabat System in Canonical Gauge*, JINR Comm. R5-87-333, Dubna 1987.

[18] Yanovski A., *Generating Operators for the Generalized Zakharov-Shabat System and its Gauge Equivalent System in* $\mathfrak{sl}(3,\mathbb{C})$ *Case*, Universität Leipzig, Naturwissenchaftlich Theoretisches Zentrum NTZ Report No 20 (1993).

[19] Yanovski A., *Geometry of the Recursion Operators*, Chapter in the book: Iliev I., Khristov E. and Kirchev K., *Spectral Methods in Soliton Equations*, Pitman Monographs and Surveys in Pure and Applied Mathematics **73**, Wiley, New York 1994.

[20] Gerdjikov V. and Yanovski A., *Completeness of the Eigenfunctions for the Caudrey-Beals-Coifman System*, J. Math. Phys. **35** (1994) 3687–3721.

[21] Bordag L. and Yanovski A., *Polynomial Lax Pairs for the Chiral* O(3)*-Field Equations and Landau-Lifshitz Equation*, J. Phys. A: Math. Gen. **28** (1995) 4007–4013.

[22] Yanovski A., *Geometric Approach to the Soliton Equations*, Universität Leipzig Naturwissenchaftlich Theoretisches Zentrum NTZ Report No 27 (1996).

[23] Bordag L. and Yanovski A., *Algorithmic Construction of* O(3) *Chiral Fields Equations Hierarchy and the Landau-Lifshitz Equation Hierarchy via Polynomial Bundle*, J. Phys. A: Math. Gen. **29** (1996) 5575–5590.

[24] Yanovski A., *Bi-Hamiltonian Formulation of the* O(3) *Chiral Fields Equations Hierarchy Obtained Through Polynomial Bundle*, Universität Leipzig, NTZ Report No 7 (1997).

[25] Yanovski A., *Bi-Hamiltonian Formulation of the Landau-Lifsitz Equation Hierarchy Obtained Through Polynomial Bundle*, Otto-Von-Guericke-Universität Magdeburg, Fakultät für Mathematik, Preprint N 48 (1997).

[26] Yanovski A., *Bi-Hamiltonian Formulation of the* O(3) *Chiral Field Equation Hierarchy Related to Polynomial Bundle*, J. Phys. A: Math. Gen. **31** (1998) 8709–8726.

[27] Yanovski A., *Linear Bundles of Lie Brackets and Their Applications*, J. Math. Phys. **41** (2000) 7869–7882.

[28] Yanovski A., *Bundles of Lie Algebras and Compatible Poisson Brackets*, Int. J. Mod. Phys. A **17** (2002), 946–950.

[29] Yanovski A., *"Universal" Lie Algebra Extensions and Their Casimir Invariants via Commutative Structures*, Physica D **172** (2002) 30–48.

[30] Yanovski A., *Extensões Universáis e Algebras Associativas*, Proceedings of the VII Encontro Sergipano de Física 2002, pp. 49–50.

[31] Yanovski A., *The Phase Space for Conformal Particle with Zero Mass and Spirality and Its Relation with the Kepler's Problem*, J. Geom. Symmetry Phys. **3** (2005) 84–104.

[32] Yanovski A., *Recursion Operators and Bi-Hamiltonian Formulations of the Landau-Lifshitz Equation Hierarchies*, J. Phys. A: Math. Gen. **39** (2006) 2409–2433.

[33] Yanovski A., *Compatible Poisson Tensors Related to Bundles of Lie Algebras*, Geom. Integr. Quantization **7** (2005) 307–319.

[34] Gerdjikov V., Vilasi G. and Yanovski A., *Integrable Hamiltonian Hierarchies. Spectral and Geometric Methods*, Lecture Notes in Physics 748, Springer, Berlin 2008.

[35] Yanovski A. and Dos Santos M., *Quadratic Casimir Invariants for "Universal" Lie Algebra Extensions*, AIP Conf. Proc. **1340** (2011), 165–179.

[36] Yanovski A., *Poisson-Nijenhuis Structures for Generalized Zakharov-Shabat System in pole Gauge on the Algebra* $\mathfrak{sl}(3,\mathbb{C})$, Geom. Integr. Quantization **12** (2012) 342–353.

[37] Yanovski A., *Geometric Interpretation of the Recursion Operators for the Generalized Zakharov-Shabat System in Pole Gauge on the Algebra* $\boldsymbol{A}_2$, J. Geom. Symmetry Phys. **23** (2011) 97–111.

[38] Yanovski A., *On the Recursion Operators for the Gerdjikov, Mikhailov and Valchev System*, J. Math. Phys. **52** (2011) 082703.

[39] Yanovski A. and Vilasi G., *Geometric Theory of the Recursion Operators for the Generalized Zakharov-Shabat System in Pole Gauge on the Algebra* $\mathfrak{sl}(n,\mathbb{C})$ *with and without Reductions*, SIGMA **8** (2012) 087, 23 pp.

[40] Yanovski A., *Geometry of the Recursion Operators for Caudrey-Beals-Coifman System in the Presence of Mikhailov* $\mathbb{Z}_p$ *Reductions*, J. Geom. Symmetry Phys.) **25** (2012) 77–97.

[41] Yanovski A. and Vilasi G., *Geometry of the Recursion Operators for the GMV System*, J. Nonlin. Math. Phys. **19** (2012) 1250023.

[42] Yanovski A., *Expansions over Adjoint Solutions for the Caudrey-Beals-Coifman System with Reductions of Mikhailov Type*, Geom. Integr. Quantization **14** (2013) 253–267.

[43] Yanovski A., *Recursion Operators and Expansions over Adjoint Solutions for the Caudrey-Beals-Coifman System with $\mathbb{Z}_p$ Reductions of Mikhailov type*, J. Geom. Symmetry Phys. **30** (2013) 105–120.

[44] Gerdjikov V. and Yanovski A., *On Soliton Equations with $\mathbb{Z}_h$ and $\mathbb{D}_h$ Reductions: Conservation Laws and Generating Operators*, J. Geom. Symmetry Phys. **31** (2013) 57–92.

[45] Gerdjikov V. and Yanovski A., *Riemann-Hilbert Problems, Families of Commuting Operators and Soliton Equations*, J. Phys: Conf. Series **482** (2014) 012017.

[46] Yanovski A., *Locality of the Conservation Laws for the Soliton Equations Related to Caudrey-Beals-Coifman System*, J. Geom. Symmetry Phys. **33** (2014) 91–107.

[47] Gerdjikov V. and Yanovski A., *CBC Systems with Mikhailov Reductions by Coxeter Automorphism: I. Spectral Theory of the Recursion Operators*, Stud. Appl. Math. **134** (2015) 145–180.

[48] Yanovski., *Some Properties of the Spectral Decompositions for the Recursion Operators Related to the Caudrey-Beals-Coifman System in the Presence of $\mathbb{Z}_p$ Reductions of Mikhailov type*, In: *Mathematics in Industry*, A. Slavova (Ed), Cambridge Scholars Publishing 2014, pp. 321–334.

[49] Yanovski A., *Book Review: "Lie-Bäcklund-Darboux Transformations" by Charles Li and Artyom Yurov*, J. Geom. Symmetry Phys. **35** (2014) 115–118.

[50] Yanovski A., *Recursion Operators for Rational Bundle on $\mathfrak{sl}(3,\mathbb{C})$ with $\mathbb{Z}_2\times\mathbb{Z}_2\times\mathbb{Z}_2$ Reduction of Mikhailov Type*, Geom. Integr. Quantization **16** (2015) 301–311.

[51] Yanovski A., *Gauge Equivalent Integrable Equations on $\mathfrak{sl}(3)$ with Reductions*, J. Phys.: Conf. Ser. **621** (2015) 012017.

[52] Yanovski A., *Gauge-Covariant Theory of the Generating Operators Associated with Linear Problems of Caudrey-Beals-Coifman Type in Canonical and in Pole Gauge with and without Reductions*, BGSIAM'14 Proceedings, A. Slavova (Ed), Sofia 2015, pp. 2–43.

[53] Yanovski A., *Some Aspects of the Spectral Theory for $\mathfrak{sl}(3,\mathbb{C})$ System with $\mathbb{Z}_2\times\mathbb{Z}_2\times\mathbb{Z}_2$ Reduction of Mikhailov Type with General Position Boundary Conditions*, Geom. Integr. Quantization **17** (2016) 379–391.

[54] Yanovski A. and Vilasi G., *Recursion Operators for CBC System with Reductions. Geometric Theory*, Il Nuovo Cimento C **38** (2015) 172.

[55] Yanovski A., *Spectral Theory of* $\mathfrak{sl}(3,\mathbb{C})$ *Auxiliary Linear Problem with* $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ *Reduction of Mikhailov Type*, In: K. Georgiev, M. Todorov and I. Georgiev (Eds), Advanced Computing in Industrial Mathematics, Studies in Computational Intelligence **681**, Springer, Berlin 2017, pp. 251–262.

[56] Valchev T. and Yanovski A., *New Reductions of a Matrix Generalized Heisenberg Ferromagnet Equation*, Pliska Stud. Math. **29** (2018) 179–188.

[57] Yanovski A. and Valchev T., *Pseudo-Hermitian Reduction of a Generalized Heisenberg Ferromagnet Equation. I. Auxiliary System and Fundamental Properties*, J. Nonl. Math. Phys. **25** (2018) 324–350.

[58] Valchev T. and Yanovski A., *Pseudo-Hermitian Reduction of a Generalized Heisenberg Ferromagnet Equation. II. Special Solutions*, J. Nonl. Math. Phys. **25** (2018) 442–461.

[59] Yanovski A. and Valchev T., *Hermitian and Pseudo-Hermitian Reduction of the GMV Auxiliary System. Spectral Properties of the Recursion Operators*, In: *Advanced Computing in Industrial Mathematics. BGSIAM 2017*, Studies in Computational Intelligence **793**, Springer 2019, pp. 433–446.

[60] Yanovski A., *Hierarchies of Symplectic Structures for* $\mathfrak{sl}(3,\mathbb{C})$ *Zakharov-Shabat Systems in Canonical and Pole Gauge with* $\mathbb{Z}_2 \times \mathbb{Z}_2$ *Reduction of Mikhailov Type*, Geom. Integr. Quantization **20** (2019) 297–310.

[61] Valchev T. and Yanovski A., *Solutions to a Vector Heisenberg Ferromagnet Equation Related to Symmetric Spaces*, Geom. Integr. Quantization **20** (2019) 285–296.

Additional References

[62] Ablowitz M., Kaup D., Newell A. and Segur H., *The Inverse Scattering Transform – Fourier Analysis for Nonlinear Problems*, Stud. Appl. Math. **53** (1974) 249–315.

[63] Beals, R. and Coifman, R., *Scattering and Inverse Scattering for First Order Systems*, Comm. Pure Appl. Math. **37** (1984) 39–90.

[64] Gerdjikov V., *Algebraic and Analytic Aspects of N-wave type Equations*, Contemporary Mathematics **301** (2002) 35–68.

[65] Gerdjikov V., *Basic Aspects of Soliton Theory*, Geom. Integr. Quantization **6** (2005) 78–125.

[66] Gerdjikov V., Mikhailov A. and Valchev T., *Reductions of Integrable Equations on **A.III**-Symmetric Spaces*, J. Phys. A **43** (2010) 434015.

[67] Mikhailov A., *The Reduction Problem and the Inverse Scattering Problem*, Physica D **3** (1981) 73–117.

[68] Mikhailov A., Olshanetsky M. and Perelomov A., *Two-Dimensional Generalized Toda Lattice*, Commun. Math. Phys. **79** (1981) 473–488.

[69] Novikov S., Manakov S., Pitaevskii L. and Zakharov V., *Theory of Solitons*: *the Inverse Scattering Method*, Plenum Press/Consultant Bureau, New York 1984.

[70] Shabat A., *Inverse Scattering Problem for a System of Differential Equations*, Funct. Anal. Appl. **9** (1975) 244–247.

[71] Zakharov V. and Manakov S., *The Theory of Resonant Interaction of Wave Packets in Nonlinear Media*, Sov. Phys. JETP **42** (1975) 842–850.

[72] Zakharov, V. and Shabat A., *Exact Theory of Two-Dimensional Self-Focusing and One-Dimensional Self-Modulation of Waves in Nonlinear Media*, Sov. Phys. JETP **34** (1972) 62–69.

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