NEW ENVELOPE EQUATIONS FOR SHALLOW WATER WAVES, MODULATIONAL INSTABILITY AND SOLITARY WAVES

ANDREI MARIN and ADRIAN STEFAN CARSTEA

Communicated by Andrei Ludu

The dynamics of wave groups is studied for long waves, using the framework of the Benjamin-Bona-Mahony (BBM) equation and its generalizations. It is shown that the dynamics are richer than the corresponding results obtained just from the Korteweg–de Vries-type equation. First, a reduction to a nonlinear Schrödinger equation is obtained for weakly nonlinear wave packets, and it is demonstrated that either the focusing or the defocusing case can be obtained. This is in contrast to the corresponding reduction for the Korteweg–de Vries equation, where only the defocusing case is found. The focusing regime displays modulational instability responsible for the appearance of rogue waves. Next, the condition for modulational instability is obtained in the case of one and two monochromatic waves in interaction at slow space-time coordinates with equal scalings. Other new envelope equations are obtained starting from the general system describing shallow water waves found by Bona *et al* [2]. A presumably integrable system is obtained from the integrable Kaup-Boussinesq one.

MSC: 35B35,35Qxx, 37K10, 37N10
Keywords: Bilinear formalism, multiple scales, solitons

Contents