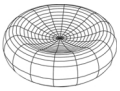
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C.C. CHEN'S MINIMAL SURFACE: FROM PARAMETRIC FORM TO ALGEBRAIC EQUATION

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Using the Weierstrass–Enneper representation, Chi Cheng Chen's complex curve in three-dimensional complex space is considered, and its real minimal surface in three-dimensional Euclidean space is studied. With the computer algebra system Maple and the Maple package `FGb` for computing Gröbner bases and solving polynomial systems, variable elimination is performed and the algebraic (or implicit) equations of Chen's complex minimal curve are computed. The algebraic minimal equation of Chen's parametric (or explicit) minimal surface in Cartesian coordinates is also calculated. Furthermore, the algebraic equation of Chen's parametric non-minimal surface in inhomogeneous tangential coordinates is computed. These computations result in determining the degree and the class of Chen's minimal surface. The surfaces are illustrated with a number of figures.

MSC: 53A10, 53C42
Keywords: Chen's minimal curve, Chen's minimal surface, complex space, Euclidean space, Weierstrass–Enneper formula

1. Introduction

Minimal surfaces in three-dimensional Euclidean space $\mathbb{E}^3$ are characterized by having zero mean curvature (i.e., $H = 0$). A comprehensive survey of minimal surfaces can be found in the book by Gray *et al* [10].

Lie [17] indicated that every algebraic minimal curve can generate a real algebraic minimal surface. Several works on algebraic minimal surfaces are available, including those by Ribaucour [20], Henneberg [15, 16], Richmond [21–23], Güler [11–14].

Further studies on classical minimal surfaces can be found in the works of Toda and Athukorallage [24], Aulisa *et al* [2], and Atampalage *et al* [1].

Barbosa and Colares [3] presented only the Weierstrass–Enneper pair, also referred to on short as Weierstrass pair, $(\omega^2, \omega + \omega^{-1})$, where $\omega \in \mathbb{C}$, for C.C. Chen's real minimal surface, and they remarked that the surface is complete.