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ON THE GEOMETRY OF $\mathbb{SO}(3)$

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The geometry of the rotation group $\mathbb{SO}(3)$ is of interest in many areas of mathematics and mechanics, and many papers are devoted to the study of this group. In the first part of this paper, the geometry of a known submersion on the rotation group with a base on a two-dimensional sphere is studied. It is proved that this submersion is Riemannian and generates a totally geodesic Riemannian foliation. In the second part of the paper, the geometry of the Liouville foliation on the cotangent bundle $T^*\mathbb{SO}(3)$ of the group $\mathbb{SO}(3)$ generated by a completely integrable Hamiltonian system is studied. It is shown that the regular leaf of this foliation is a three-dimensional surface of non-zero normal curvature and zero Gaussian torsion.

MSC: 53C12, 57R30
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1. Introduction

Let M be a smooth Riemannian manifold of dimension n with the Riemannian metric g, ∇ be the Levi-Civita connection, and $\langle\cdot,\cdot\rangle$ be the inner product defined by the Riemannian metric g.

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