



EULER'S ELASTICA IN NONLOCAL THEORY OF ELASTICITY

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A generalization of the Euler's elastic problem, i.e., finding a stationary configuration (planar elastica) of the Bernoulli's thin ideal elastic rod with boundary conditions defined through fixed endpoints and/or tangents at the endpoints, for the chosen nonlocal differential constitutive stress-strain relation (i.e., nonlocal theory of elasticity) is considered. In the classical (local) Euler-Bernoulli's beam model, the general solutions of the governing equations (that are inhomogeneous but linear) for bending moments and shear forces in the case of large deformations can be obtained using the Jacobi elliptic functions and incomplete elliptic integrals. For the discussed nonlocal toy differential model, the general solutions of the governing equations (that are this time nonlinear) can also be expressed in the parametric form through the linear combinations of all three incomplete elliptic integrals. As further research, we plan to apply some boundary conditions (clamped, simply supported, etc.) for the obtained nonlocal general solutions in order to compare them to the local solutions for the corresponding boundary value problems.

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Contents

1	Introduction	24
2	Explicit Analytical Solutions for Local Euler's Elastica	24
3	Toy Differential Model for Nonlocal Euler's Elastica	28
4	Implicit Parametrization Through Incomplete Elliptic Integrals	31
5	Final Remarks	34
	References	34
	doi: 10.7546/jgsp-74-2025-23-37	23