



# CONSERVATION OF ENERGY, ENVELOPES AND GEODESICS ON SURFACES OF REVOLUTION

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We consider the Newton equations and the Euler–Lagrange equations that admit reduction of order via the conservation of energy law. Passing to the first-order equation of the constant energy, “extraneous solutions” that are not solutions of the initial second-order equation can appear. Analysis of “extraneous solutions” leads to a number of interesting results. For example, it helps to understand the behaviour of geodesics on surfaces of revolution.

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## 1. Introduction

This tutorial note may be useful for those who teach a course in ordinary differential equations and want to make it more interesting, geometric, and related to other sections of mathematics and physics. From a methodological viewpoint, it