



SOURIAU-FISHER METRIC AND COMPLETELY INTEGRABLE SYSTEM ON LIE GROUPS $SO(2)$ AND $SO(3)$

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Communicated by Akira Yoshioka

We study the generalized Fisher metric on the Lie groups $SO(2)$ and $SO(3)$ via the Souriau thermodynamics Lie group theories. Then we give the effect of two-cycle on the integrability of gradient systems due to the Fisher metric and the Souriau-Fisher metric. In addition, we show how the cocycle can locally modify the Fisher metric on a coadjoint orbit, in explicit terms of brackets and central extensions on the Lie groups $SO(2)$ and $SO(3)$.

MSC: 20P05, 51H20, 51H25, 53D05, 53Z05, 53Z30, 62E10

Keywords: Coadjoint orbits, cocycle, Fisher metric, geometric thermodynamics, Hamiltonian system, integrability, Lie group

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	doi: 10.7546/jgsp-76-2026-1-23	1