



SQUARE ROOTS OF REAL AND COMPLEX QUATERNIONS

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Square roots of real and complex (complexified) quaternions, namely, the Hamilton’s quaternion, coquaternion, nectorine, and conectorine are investigated. The isomorphism between the quaternions and multivectors in Clifford algebras is employed for this purpose. Root examples for all named quaternions are presented from which follows that the real and complex quaternionic roots may assume multiple discrete or continuous forms, or there may be no roots at all.

MSC: 15A18, 15A66

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