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BI-HAMILTONIAN STRUCTURES RELATED TO THE EIGENVALUE PROBLEMS GENERATING LIE ALGEBRA

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In this article we will show the correspondence between eigenvalue problems defining some Lie algebra and the possibility to generate a compatible Lie brackets. Compatible Lie brackets give rise to a bi-Hamiltonian structure, which can be effectively used to construct integrable systems related to this Lie algebra. We will describe this construction from the perspective of eigenvalue problems and illustrate it on selected example.

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Contents

1. Introduction

The concept of a bi-Hamiltonian structure is a very powerful tool in the theory of integrable systems. It was first introduced in Magri's paper [13] on the construction of the complete sequence of first integrals for the Korteweg–de Vries equation. Therefore, we have a manifold M equipped with two compatible Poisson brackets $\{\cdot,\cdot\}_1$, $\{\cdot,\cdot\}_2$. This bi-Hamiltonian manifold is often denoted by $(M, \{\cdot,\cdot\}_1, \{\cdot,\cdot\}_2)$. The Poisson bracket compatibility means that their linear combination

$$\{\cdot,\cdot\}_\lambda = \{\cdot,\cdot\}_1 + \lambda\{\cdot,\cdot\}_2, \qquad \lambda \in \mathbb{R}$$

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