



GEOMETRY OF LIOUVILLE FOLIATION FOR THE INTEGRABLE CASE OF THE HÉNON–HEILES SYSTEM

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The geometric study of invariant manifolds in completely integrable Hamiltonian systems remains a fundamental topic in mathematical physics. This paper investigates the Sawada–Kotera case of the generalized Hénon–Heiles system on a four-dimensional Poisson manifold. We focus on the metric properties of the Liouville tori that foliate the phase space. By analyzing the momentum map and its critical values, we explicitly compute the components of the Gaussian torsion. Our main result demonstrates that the Gaussian torsion vanishes identically across all regular chambers of the bifurcation diagram, identifying these tori as a geometrically distinguished class of surfaces.

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