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ON THE FIRST PONTRYAGIN FORM OF A SURFACE

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Abstract. The first Pontryagin form of a compact orientable surface M determines a canonical pre-symplectic structure on the space of Riemannian metrics on M. The first equivariant Pontryagin form determines a canonical moment map for it. We study the corresponding symplectic reduction and we state (Theorem 6) that the symplectic quotient is the Teichmüller space of the surface with the Weil-Petersson symplectic form.

1. Introduction

The universal Pontryagin forms are differential forms on the first jet-bundle of the bundle of Riemannian metrics of a manifold M. They are closed and invariant under the natural action of the diffeomorphism group of M. Pulling-back a universal Pontryagin form by means of the first jet prolongation j^1g of a Riemmanian metric g we obtain a closed differential form on M. As the space of Riemannian metrics is contractible, the maps j^1g are homotopic for different metrics g, and hence the cohomology class of the form obtained on M is independent of the metric g chosen. This cohomology class is the corresponding Pontryagin class of M. In this way we recover the result that the Pontryagin classes are independent of the Riemannian metric used to construct them.

Moreover, with this construction we obtain more than merely the Pontryagin classes. The key point is the fact that *there are non-zero universal Pontryagin forms of degree greater than the dimension of M*. We show that these higher-order universal Pontryagin forms can be interpreted geometrically. Concretely, we study the interpretation of the first universal Pontryagin form of a compact and oriented surface. In addition, the equivariant Pontryagin forms are also introduced, thus providing canonical equivariant extensions of the universal Pontryagin forms.

By applying the results of [6] to the case of Riemannian metrics we obtain that the first universal Pontryagin form of a compact and oriented surface determines a