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ANALYTIC DESCRIPTION OF THE VISCOUS FINGERING INTERFACE IN A ROTATING HELE-SHAW CELL

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Abstract. The determination of the interface shape between two fluids during the process of viscous fingering (Saffman–Taylor instability) in a Hele-Shaw cell is addressed here. The parametric equations describing this interface are obtained in an explicit analytic form. With these results, a number of interface shapes are presented, including shapes with opposite sides in contact that are considered as the onset of droplet pinch-off.

1. Introduction

Viscous fingering (Saffman–Taylor instability [8]) is the formation of patterns in the interface between two fluids in a Hele-Shaw cell. It occurs during injection when a less viscous fluid displaces a more viscous one in planar or non-planar Hele-Shaw cell [7]. It can also occur due to gravity with or without taking into account chemical or thermal effects [1, 9] if a horizontal interface separates two fluids of different densities and the heavier fluid is above the other one. A closely related problems appear in the study of the collapse of nanotubes [10] and rings exposed to uniform externed presure [2].

Saffman–Taylor instability also occurs in many other frameworks, e.g. in a Hele–Shaw cell subjected to pressure, radial magnetic field or rotation.

Let the interface be given by means of the coordinates $x(s)$, $z(s)$ in a certain Cartesian coordinate frame in the Euclidean plane with s being the interface arclength. The unit **tangent vector** $\mathbf{t}(s)$ and the unit **normal vector** $\mathbf{n}(s)$ are related to the