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COHOMOGENEITY TWO RIEMANNIAN MANIFOLDS OF NON-POSITIVE CURVATURE

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Abstract. We consider a Riemannian manifold M ($\dim M \geq 3$), which is flat or has negative sectional curvature. We suppose that there is a closed and connected subgroup G of $\mathrm{Iso}(M)$ such that $\dim(M/G) = 2$. Then we study some topological properties of M and the orbits of the action of G on M.

1. Introduction

Let M^n be a connected and complete Riemannian manifold of dimension n, and G be a closed and connected subgroup of the Lie group of all isometries of M. If $x \in M$ then we denote by $G(x) = \{gx \,;\, g \in G\}$ the orbit containing x.

If $\max\{\dim G(x) \,;\, x \in M\} = n - k$, then M is called a $\boldsymbol{C_k}$**-$\boldsymbol{G}$-manifold** (G-manifold of cohomogeneity k) and we will denote it by $\mathrm{Coh}(G, M) = k$. If M is a C_k-G-manifold then the orbit space $M/G = \{G(x) \,;\, x \in M\}$ is a topological space of dimension k. When k is small, we expect close relations between topological properties of M and the orbits of the action of G on M. If M is a C_0-G-manifold then the action of G on M is transitive, so M is a homogeneous G-manifold and it is diffeomorphic to G/G_x (where $x \in M$ and $G_x = \{g \in G \,;\, gx = x\}$). Thus, topological properties of homogeneous G-manifolds are closely related to Lie group theory. If M is a homogeneous G-manifold of non-positive curvature, it is diffeomorphic to $\mathbb{R}^{n_1} \times \mathbb{T}^{n_2}$, $n_1 + n_2 = n$ ([20]). The study of C_1-G-manifolds goes back to 1957 and a paper due to Mostert [14]. Mostert characterized the orbit space of C_1- G-manifolds, when G is compact. Later, other mathematicians generalized the Mostert's theorem to G-manifolds with non-compact G. There are many interesting results on topological properties of the orbits of C_1-G-manifolds under conditions on the sectional curvature of M. If M is a C_1-G-manifold of negative curvature then it is proved (see [17]) that either M is simply connected or the fundamental group of M is isomorphic to $\mathbb{Z}^p$ for some