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MULTIPARAMETER CONTACT TRANSFORMATIONS

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Abstract. This is a review of multiparameter families of contact transformations and their relationship with the generalized Hamiltonian system. We derive the integrability conditions for the generalized Hamiltonian system and show that when they are satisfied the solutions to this system determine a family of multiparameter contact transformations of the initial conditions. We prove a necessary and sufficient condition for a multiparameter family of contact transformations to be a group and a characterization of the function which describes the group multiplication rule.

1. Introduction

Let us begin by recalling a few facts about one parameter contact transformations. Consider transformations of the (x, y, z, p, q)-space to the (X, Y, Z, P, Q)-space defined by $X = X(x, y, z, p, q)$, $Y = Y(x, y, z, p, q)$, $Z = Z(x, y, z, p, q)$, $P = P(x, y, z, p, q)$, $Q = Q(x, y, z, p, q)$.

Definition 1. *Let T be a one-to-one, onto, continuously differentiable transformation of the (x, y, z, p, q)-space to the (X, Y, Z, P, Q)-space with a nonzero Jacobian. Then T is called a contact transformation if $p\,\mathrm{d}x + q\,\mathrm{d}y - \mathrm{d}z = 0$ implies $P\,\mathrm{d}X + Q\,\mathrm{d}Y - \mathrm{d}Z = 0$.*

Theorem 1. *The one-to-one, onto, continuously differentiable transformation T of the (x, y, z, p, q)-space to the (X, Y, Z, P, Q)-space with a nonzero Jacobian is a contact transformation if and only if there exists a nonzero function $\rho = \rho(x, y, z, p, q)$ such that*

$$P\,\mathrm{d}X + Q\,\mathrm{d}Y - \mathrm{d}Z = \rho(p\,\mathrm{d}x + q\,\mathrm{d}y - \mathrm{d}z). \tag{1}$$