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ON THE JACOBIAN GROUP FOR MÖBIUS LADDER AND PRISM GRAPHS

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Abstract. The notion of the Jacobian group of graph (also known as Picard group, sandpile group, critical group) was independently given by many authors. This is a very important algebraic invariant of a finite graph. In particular, the order of the Jacobian group coincides with the number of spanning trees for a graph. The latter number is known for the simplest families of graphs such as Wheel, Fan, Prism, Ladder and Möbius ladder graphs. At the same time the structure of the Jacobian group is known only in several cases. The aim of this paper is to determine the structure of the Jacobian group of the Möbius ladder and Prism graphs.

1. Introduction

We define a Möbius ladder M_n of order n as the cubic circulant graph $C_{2n}(1, n)$ with $2n$ vertices. In this case, M_n can be considered as a regular $2n$-gon whose n pairs of opposite vertices are joint by an edge. One can also realize M_n as a ladder with n steps on the Möbius band. A **Prism graph** Pr_n, sometimes also called a **circular ladder graph**, is a cubic graph with $2n$ vertices, which are connected as shown in Fig. 1. Notice, that Prism graph Pr_{2n} is a double cover of Möbius ladder M_n. It is a discrete version of the statement that the cylinder is a double cover of the Möbius band.

The aim of the present paper is to find the structure of the Jacobian group of the Möbius ladder M_n and Prism graph Pr_n.

The notion of the Jacobian group of a graph (also known as Picard group, sandpile group, critical group) was independently given by many authors [1–3, 5]. This is