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SYMMETRY REDUCTION OF ASYMMETRIC HEAVENLY EQUATION AND 2+1-DIMENSIONAL BI-HAMILTONIAN SYSTEM

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Abstract. Asymmetric heavenly equation, presented in a two-component form, is known to be 3+1-dimensional bi-Hamiltonian system. We show that symmetry reduction of this equation yields a new two component 2+1-dimensional integrable bi-Hamiltonian system. We prove that this new 2+1-dimensional system admits bi-Hamiltonian structure, so that it is integrable according to Magri's theorem.

1. Introduction

Asymmetric heavenly equation was obtained as one of the canonical equations in the classification of nonlinear second order partial differential equations that possess partner symmetries [1]. The asymmetric heavenly equation in 3+1-dimension is given by

$$u_{tx}u_{ty} - u_{tt}u_{xy} + au_{tz} + bu_{xz} + cu_{xx} = 0 \tag{1}$$

where u is the unknown function that depends on the four independent variables t, x, y, z and the subscripts denote partial derivatives of u, e.g., $u_{tx} = \partial^2 u/\partial t \partial x$, $u_{xx} = \partial^2 u/\partial x^2$..., while a, b, c are constants. By choosing $u_t = v$ as the second unknown, we have converted the asymmetric heavenly equation to the two-component evolution system [2]

$$u_t = v, \qquad v_t = \frac{1}{u_{xy}}\big(v_x v_y + av_z + bu_{xz} + cu_{xx}\big) \equiv Q. \tag{2}$$

The physical significance of the singe scalar equation (1) follows from the fact that it is equivalent to complex Einstein field equations for (anti-)self-dual gravitational fields [3], with u being the metric potential.