BERTRAND SYSTEMS ON SPACES OF CONSTANT SECTIONAL CURVATURE. THE ACTION-ANGLE ANALYSIS. CLASSICAL, QUASI-CLASSICAL AND QUANTUM PROBLEMS

JAN JERZY SŁAWIANOWSKI and BARBARA GOŁUBOWSKA

Institute of Fundamental Technological Research, Polish Academy of Sciences
5^B Pawińskiego Str., 02-106 Warsaw, Poland

Abstract. Studied is the problem of degeneracy of mechanical systems the configuration space of which is the three-dimensional sphere, the elliptic space, i.e., the quotient of that sphere modulo the antipodal identification, and finally, the three-dimensional pseudo-sphere, namely, the Lobatchevski space. In other words, discussed are systems on groups $SU(2)$, $SO(3,\mathbb{R})$, and $SL(2,\mathbb{R})$ or its quotient $SO(1,2)$. The main subject are completely degenerate Bertrand-like systems. We present the action-angle classical description, the corresponding quasi-classical analysis and the rigorous quantum formulas. It is interesting that both the classical action-angle formulas and the rigorous quantum mechanical energy levels are superpositions of the flat-space expression, with those describing free geodetic motion on groups.

MSC: 51P05, 53A35, 37N05
Keywords: action-angle description, Bertrand systems, completely degenerate problems, elliptic space, Lobatchevski space, quasi-classical analysis, sphere

CONTENTS