

GEOMETRIC MODELS FOR SECONDARY STRUCTURES IN PROTEINS

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Abstract. This research is motivated by a study of special types of surfaces of revolution, using methods from differential geometry, elasticity theory and variational calculus. In particular, we present an elastic membrane model for the beta barrels in protein biology, via a certain *Generalized Willmore type* energy functional. We study the corresponding Euler–Lagrange equation, as well as a specific boundary value problem whose solutions are *Generalized Willmore surfaces of revolution*. We study the corresponding solutions both theoretically and numerically.

MSC: 53A05, 53A10, 74B20

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