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GENERALIZED EULER ANGLES VIEWED AS SPHERICAL COORDINATES

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Abstract. Here we develop a specific factorization technique for rotations in $\mathbb{R}^3$ into five factors about two or three fixed axes. Although not always providing the most efficient solution, the method allows for avoiding gimbal lock singularities and decouples the dependence on the invariant axis $\mathbf{n}$ and the angle ϕ of the compound rotation. In particular, the solutions in the classical Euler setting are given directly by the angle of rotation ϕ and the coordinates of the unit vector $\mathbf{n}$ without additional calculations. The immediate implementations in rigid body kinematics are also discussed and some generalizations and potential applications in other branches of science and technology are pointed out as well.
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Introduction

Euler angles and their variants are well-studied tool for the representation of the three-dimensional rotation group and its spin cover $\mathrm{SU}(2) \cong \mathbb{S}^3$, which find numerous applications in both classical and quantum mechanics. There are twelve known configurations, in which the compound rotation is decomposed into three