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CAYLEY MAP AND HIGHER DIMENSIONAL REPRESENTATIONS OF ROTATIONS

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Abstract. The embeddings of the $\mathfrak{so}(3)$ Lie algebra and the Lie group SO(3) in higher dimensions is an important construction from both mathematical and physical viewpoint. Here we present results based on a program package for building the generating matrices of the irreducible embeddings of the $\mathfrak{so}(3)$ Lie algebra within $\mathfrak{so}(n)$ in arbitrary dimension $n \geq 3, n \neq 4k+2, k \in \mathbb{N}$ relying on the algorithm developed recently by Campoamor-Strursberg [3]. For the remaining cases $n = 4k + 2$ embeddings of $\mathfrak{so}(3)$ into $\mathfrak{so}(n)$ are also constructed. Besides, we investigate the characteristic polynomials of these $\mathfrak{so}(n)$ elements. We show that the *Cayley* map applied to $\mathcal{C} \in \mathfrak{so}(n)$ is well defined and generates a subset of SO(n). Furthermore, we obtain explicit formulas for the images of the *Cayley* map. The so obtained SO(n) matrices are expressed as polynomials of $\mathcal{C}$ whose coefficients are rational functions of the norm of the vector-parameter **c**. The composition laws are extracted for the cases $n = 4, 6$ and for the first case it is shown that via the *Cayley* map the isomorphism SU$(2) \cong \operatorname{im} \mathrm{Cay}_{\operatorname{im} j_4} \cup \{-\mathcal{I}_4\}$ holds. Also, for $n = 4$ explicit formulas for the the angular velocity matrices are derived. Comparisons between the results obtained via the exponential map and the *Cayley* map are made as well. In contrast to the case of the *Cayley* map, the results for the exponential map include either irrational or transcendental functions of the module of the vector-parameter.

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