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QUANTIZATION OF LOCALLY SYMMETRIC KÄHLER MANIFOLDS

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Abstract. We introduce noncommutative deformations of locally symmetric Kähler manifolds. A Kähler manifold M is said to be a locally symmetric Kähler manifold if the covariant derivative of the curvature tensor is vanishing . An algebraic derivation process to construct a locally symmetric Kähler manifold is given. As examples, star products for noncommutative Riemann surfaces and noncommutative $\mathbb{CP}^N$ are constructed.

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1. Review of the Deformation Quantization with Separation of Variables

In this section, we review the deformation quantization with separation of variables to construct noncommutative Kähler manifolds.

An N-dimensional Kähler manifold M is described by using a Kähler potential. Let Φ be a Kähler potential and ω be a Kähler two-form

$$\omega := \mathrm{i} g_{k\bar{l}} \mathrm{d}z^k \wedge \mathrm{d}\bar{z}^l, \qquad g_{k\bar{l}} := \frac{\partial^2 \Phi}{\partial z^k \partial \bar{z}^l} \tag{1}$$

where $z^i, \bar{z}^i$ $(i = 1, 2, \ldots, N)$ are complex local coordinates.

In this article, we use the Einstein summation convention over repeated indices. The $g^{\bar{k}l}$ is the inverse of the Kähler metric tensor $g_{k\bar{l}}$. That means $g^{\bar{k}l} g_{l\bar{m}} = \delta_{\bar{k}\bar{m}}$.