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INTEGRABILITY OF THE TWO-LAYER SPIN SYSTEM

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Abstract. Among nonlinear evolutionary equations integrable ones are of particular interest since only in this we case can theoretically study the model in detail and in-depth. In the present, we establish the geometric connection of the well-known integrable two-component Manakov system with a new two-layer spin system. This indicates that the latter system is also integrable. In this formalism, geometric invariants define some important conserved quantities associated with two interacting curves, and with the corresponding nonlinear evolution equations.

MSC: 53C05, 53C35
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1. Introduction

The first and the most known representative of the integrable spin systems is the (1+1)-dimensional Heisenberg ferromagnetic equation (HFE) which has a form

$$\mathbf{S}_t = \mathbf{S} \times \mathbf{S}_{xx} \tag{1}$$

where $\mathbf{S} = (S_1, S_2, S_3)$ is a unit spin vector, $\mathbf{S}^2 = 1$ and matrix form of the vector $\mathbf{S}$ is given by

$$S = \begin{pmatrix} S_3 & S^- \\ S^+ & -S_3 \end{pmatrix}, \qquad S^2 = I = \mathtt{diag}(1,1), \qquad S^{\pm} = S_1 \pm \mathrm{i}S_2. \tag{2}$$

The HFE (1) is geometrically equivalent to the nonlinear Schrödinger equation [3]

$$\mathrm{i}q_t + q_{xx} + 2|q|^2 q = 0. \tag{3}$$

Also, it is well-known that these equations are gauge equivalent to each other [9].