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THE CANHAM-HELFRICH MODEL FOR THE ELASTICITY OF BIOMEMBRANES AS A LIMIT OF MESOSCOPIC ENERGIES

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Abstract. In this paper we review some recent results concerning the variational deduction of a Canham-Helfrich model for biomembranes obtained starting from a mesoscopic model which implements the amphiphilic behavior of the lipid molecules and the head-tail connection. The two-dimensional analysis is complete while in the three-dimensional case we have partial results and open problems.

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1. Introduction

A prominent way to model biomembranes is given by shape energies of *Canham-Helfrich* type [1, 3, 4, 7, 17]. These type of energies have the general form

$$E(S) = \int_S \kappa_1 (H - H_0)^2 - \kappa_2 K \, \mathrm{d}\mathcal{H}^2 \tag{1}$$

where S denotes a smooth surface in $\mathbb{R}^3$, H and K are the mean curvature and the Gaussian curvature of S respectively, and the bending moduli κ_1, κ_2 and the spontaneous curvature H_0 are constant. Typically, $\kappa_1 > \kappa_2 > 0$ is a compatibility condition coming both from mathematical considerations and from experiments [14, 16]. The shape of the membrane is an absolute minimimizer of E among a suitable class of surfaces. We notice that, thanks to the Gauss-Bonnet's Theorem, when the spontaneous curvature is zero and the topology of S is fixed the minimization problem for the Canham-Helfrich functional reduces to the minimization problem for the very well studied *Willmore functional* [9, 13, 15]. The