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QUANTUM STOCHASTIC PRODUCTS AND THE QUANTUM CONVOLUTION

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Abstract. A quantum stochastic product is a binary operation on the space of quantum states preserving the convex structure. We describe a class of associative stochastic products, the twirled products, that have interesting connections with quantum measurement theory. Constructing such a product involves a square integrable group representation, a probability measure and a fiducial state. By extending a twirled product to the full space of trace class operators, one obtains a Banach algebra. This algebra is commutative if the underlying group is abelian. In the case of the group of translations on phase space, one gets a quantum convolution algebra, a quantum counterpart of the classical phase-space convolution algebra. The peculiar role of the fiducial state characterizing each quantum convolution product is highlighted.

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1. Introduction and Main Ideas

As is well known, operator algebras play a central role in several areas of modern theoretical physics; remarkably, e.g., in quantum mechanics, quantum field theory, non-commutative geometry and quantum statistical mechanics [15, 17]. Quantum *states*, in particular, can be regarded as *normalized positive functionals* on the C^*-algebra containing all bounded *observables* [15, 17]; i.e., the space $\mathcal{B}(\mathcal{H})$ of bounded operators on a separable complex Hilbert space $\mathcal{H}$, endowed with the usual composition of operators $(A, B) \mapsto AB$ (the algebra product) and with the adjoining map $A \mapsto A^*$ (the algebra involution).