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EXAMPLES OF FOUR- OR SIX-DIMENSIONAL SYMPLECTIC-HAANTJES MANIFOLDS AND ABOUT A RELATIONSHIP WITH RECURSION OPERATORS

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Abstract. Certain ways of characterizing integrable systems with $(1,1)$-tensor field have been investigated, so far. For example, recursion operators and Haantjes operators are known. We show that geometrical examples of four- or six-dimensional symplectic Haantjes manifolds and recursion operators for several Hamiltonian systems. Through these examples, we consider the relation between recursion operators and Haantjes operators.

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1. Introduction

Liouville proved that when a system with n degrees of freedom on a $2n$-dimensional phase space has n independent first integrals in involution the system is integrable by quadratures (cf. [1]).

On the other hand, S. De Filippo, G. Marmo, M. Salerno and G. Vilasi proposed a new characterization of completely integrable Hamiltonian systems. They gave the following theorem. Let X be a dynamical vector field on a $2n$-dimensional manifold $\mathcal{M}$. If the vector field X admits a diagonalizable mixed $(1,1)$-tensor field T which is invariant under X, has a vanishing Nijenhuis torsion and has doubly degenerate eigenvalues with nowhere vanishing differentials, then there exist a symplectic structure and a Hamiltonian function H such that the vector field X is separable, Hamiltonian vector field of H, and H is completely integrable with respect to the symplectic structure. The above $(1,1)$-tensor field T is called